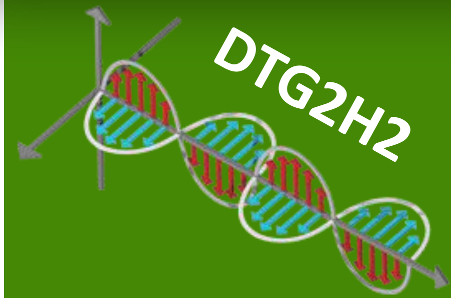


DTG2H2

ELEKTROMAGNETIK TERAPAN

1. VEKTOR ANALISIS

By Dwi Andi Nurmantris

WHERE ARE WE???

1. PENDAHULUAN
   - Pengenalan Mata Kuliah
   - Silabus (materi), dan referensi,
   - Aturan penilaian: Quis, Ujian, Tugas dll
   - Aturan Perkuliahan : Kontrak Belajar
   - Sejarah dan Aplikasi Elektromagnetika
2. PERSAMAAN MAXWELL UNTUK MEDAN DINAMIS
   - Vektor Analysis
   - Konsep dan Arti Fisis tentang Empat Persamaan Maxwell
   - penerapan konsep Empat Persamaan Maxwell
3. PROPAGASI GELOMBANG DATAR
   - Penurunan Pers. Helmholtz dari Persamaan Maxwell
   - Perambatan gelombang pada Berbagai Medium (Dielektrik Merugi)
   - Perambatan gelombang pada Dielektrik Sempurna, Vakum, Konduktor : Efek Kulit) dengan Parameter Primer dan Parameter Sekundernya
   - Vektor Poynting dan Analisis Daya
   - Polarisasi Gelombang
   - Pantulan gelombang sudut datang nol
   - Konservasi Daya dalam Pantulan
   - Standing Wave Ratio,Impedansi Input, dan Matching gelombang
   - Radome (med1|med2|med3 -med1|med2|med3)
   - Perambatan GEM pada arah sembarang
   - Pantulan Sudut-Datang Tak-Nol dan Nol : Gelombang Berdiri
4. SALURAN TRANSMISI
   - Model dan Persamaan Saluran Transmisi
   - Macam-macam Saluran Transmisi dengan Parameter Primer dan Sekundernya, Saluran Distortionless dan Lossless

WHERE ARE WE???

- Kasus 1 : Saluran Tak-merugi Beban Sesuai (V, I, P)
- Kasus 2 : Saluran Tak-merugi Beban Tak-Sesuai (V, I, P)
- Impedansi input dan VSWR
- Kasus 3 : Saluran-saluran Istimewa (λ/2, λ/4, ZL = 0, ZL = ~)
- Kasus 4 : Persoalan Saluran Merugi
- Penyesuaian Impedansi dengan Transformator ¼ panjang glb.
- Konsep lebar-pita frekuensi untuk sistem saluran transmisi
- Penyesuaian Impedansi dengan Stub-Tunggal
- Smith-Chart: Pembuatan dan Penggunaan
- Penyesuaian Impedansi dengan Stub Ganda dengan Smith Chart

5. **BUMBUNG GELOMBANG PERSEGI (BGP)**
   - Analisis Medan Elektromagnetik dalam BGP
   - Gelombang Mode TMmn, Parameter Primer dan Sekunder
   - Gelombang Mode TEmn, Parameter Primer dan Sekunder
   - Tinjauan Daya dan Rugi-rugi

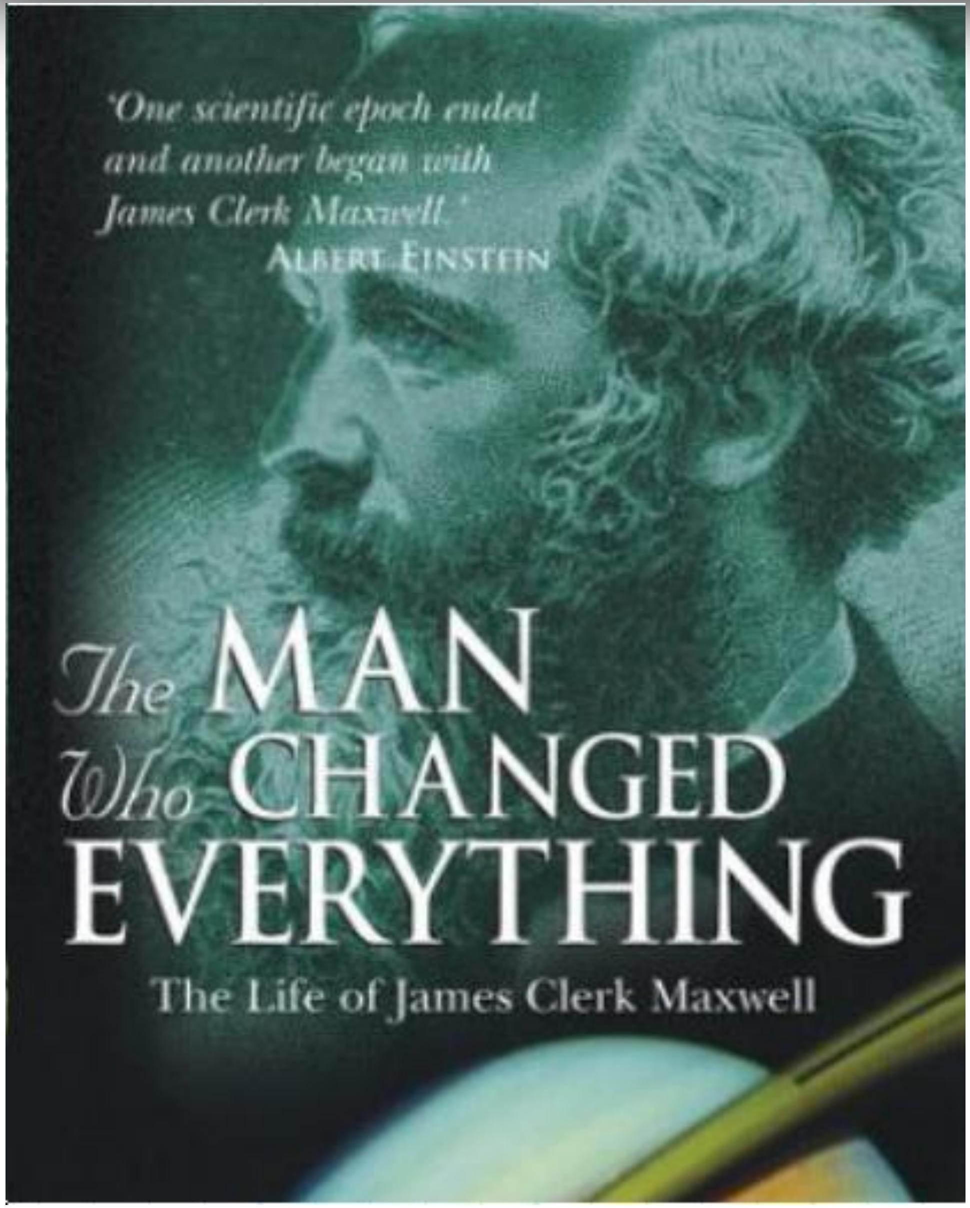
6. **BUMBUNG GELOMBANG SIRKULAR (BGS)**
   - Analisis Medan Elektromagnetik dalam BGS
   - Gelombang Mode TMnl dan TEnl , Parameter Primer dan Sekunder
   - Pengenalan Serat Optik

7. **RADIASI GELOMBANG**
   - Analisis Medan Radiasi Filamen Pendek, Diagram Arah
   - Aproksimasi untuk Medan Jauh, Daya Pancar, Tahanan Pancar Dipole ½ λ dan Monopole

OUTLINE

1. Vektor Analysis
   a) Scalar dan vektor
   b) Vektor Algebra
   c) Coordinate system
   d) Calculus of scalar and vektor (gradient, divergence, curl)
   e) Divergence Theorem
   f) Stoke's Theorem

MAXWELL'S EQUATION



MAXWELL'S EQUATION

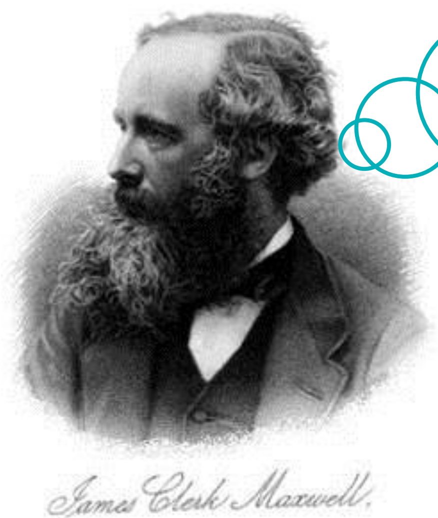
This is where we are heading

$$\nabla . \vec{D} = \rho \qquad \oint_s \vec{D}.\vec{ds} = \int_v \rho \, dv$$

$$\nabla . \vec{B} = 0 \qquad \oint_s \vec{B}.\vec{ds} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint \vec{E} . \vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t} . \vec{ds}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \qquad \int_l \vec{H} . \vec{dl} = \oint_s (\sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}) . \vec{ds}$$



DON'T PANIC!

WHY MAXWELL'S EQUATION?

- How EM field produced?
- How EM wave produced?
- How does a capasitor store energy?
- How does an antenna radiate or receive signals?
- How do Electromagnetic fields propagate in space?
- What really happens when electromagnetic energy travels from one end of waveguide to the others?

OUTLINE

1. **Vektor Analysis**
   a) Scalar and vector
   b) Vektor Algebra
   c) Coordinate system
   d) Calculus of scalar and vektor

Scalar and Vector

Skalar : Besaran yang hanya memiliki nilai

Contoh : Temperatur, laju, jarak, dll

a. **Scalar Discrete Quantities** : can be described with a single numeric value
b. **Scalar Fields Quantities** : that the magnitude must be described as one Function of (typically) space and/or time

Vektor : Besaran yang memiliki nilai dan arah

Contoh : Medan listrik, medan magnet, Gaya, kecepatan, posisi, percepatan, dll

a. **Vector Discrete Quantities** : must be described with both a single value magnitude and a direction
b. **Vector Fields Quantities:** quantities that both magnitude and/or direction must be described as a function of (typically) space and/or time

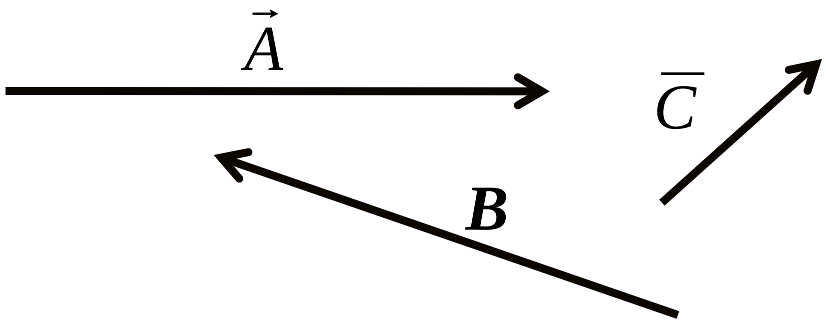
Scalar and Vector

Contoh Physical Quantity dalam scalar dan vektor

- Scalar Quantities
  - ✓ Berat
  - ✓ Tinggi
  - ✓ Temperatur
- Vector Quantities
  - ✓ Force
  - ✓ velocities
- Scalar Fields
  - ✓ Berat dalam fungsi waktu
  - ✓ Surface Temperatur saat ini pada area tertentu
- Vector Fields
  - ✓ Surface wind velocities

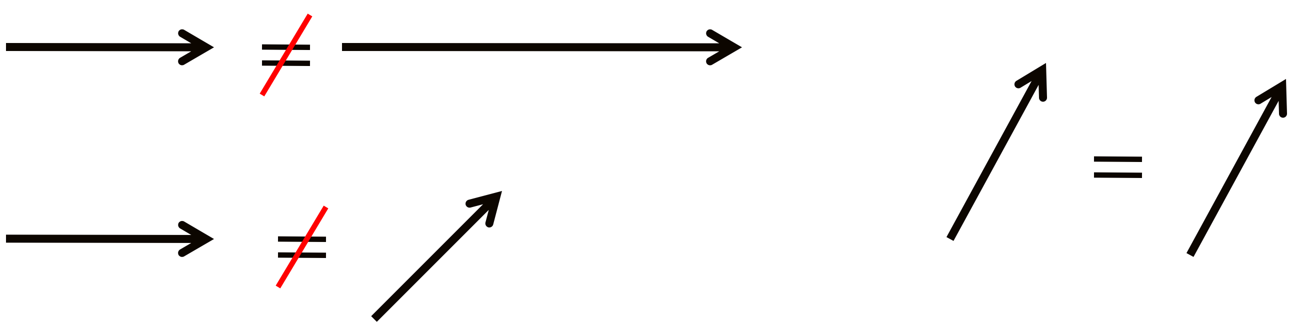
Vector Representation

symbolically we can represent a vector quantity as an **arrow**:



- The **length** of the arrow is proportional to the **magnitude** of the vector quantity.
- The **orientation of the arrow** indicates the **direction** of the vector quantity.
- The variable names of a vector quantity will always be either **boldface** or have an **overbar**

Dua buah vektor dikatakan sama jika kedua vektor tersebut memiliki magnitude dan arah yang identik



Vector Representation

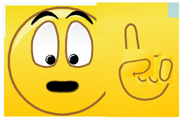
$$\vec{A} = \left|\vec{A}\right| \hat{a}_A$$

Dengan :

$\left|\vec{A}\right|$ adalah besar vektor A atau panjang vektor A (**magnitude** vektor A)

$\hat{a}_A$ adalah unit vektor A atau **vektor satuan** searah A

Vektor satuan atau unit vektor menyatakan **arah vektor**, besarnya **satu.**

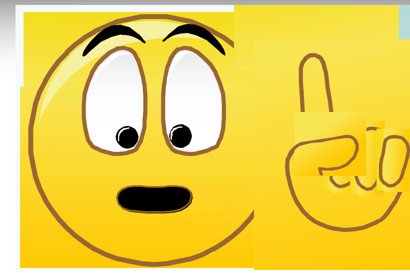


$$\hat{a}_A = \frac{\vec{A}}{\left|\vec{A}\right|}$$

$$\left|\hat{a}_A\right| = 1$$

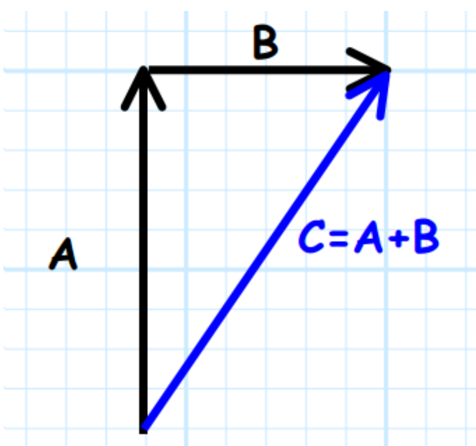
Vector Algebra

If we know the rules of vector operations, we can analyze, manipulate, and simplify vector operations!

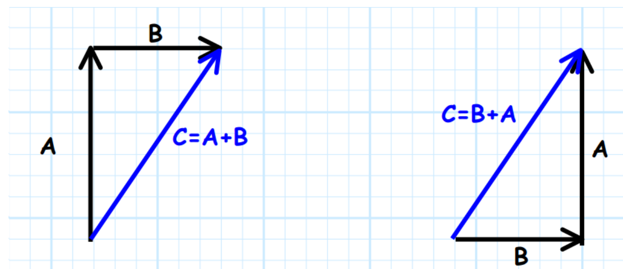


Vector Addition

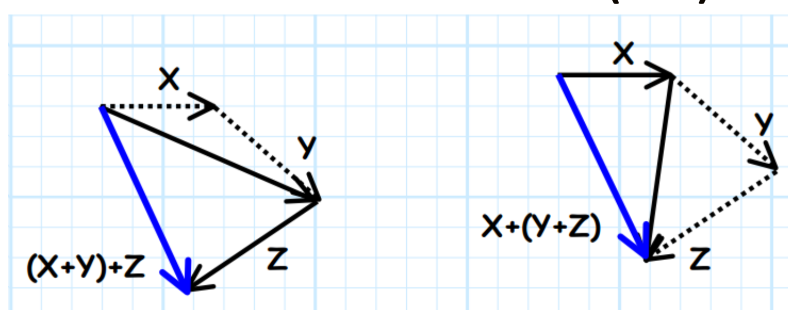
$$\vec{A}+\vec{B}=\vec{C}$$



1. Vector addition is **commutative-> A+B = B+A**

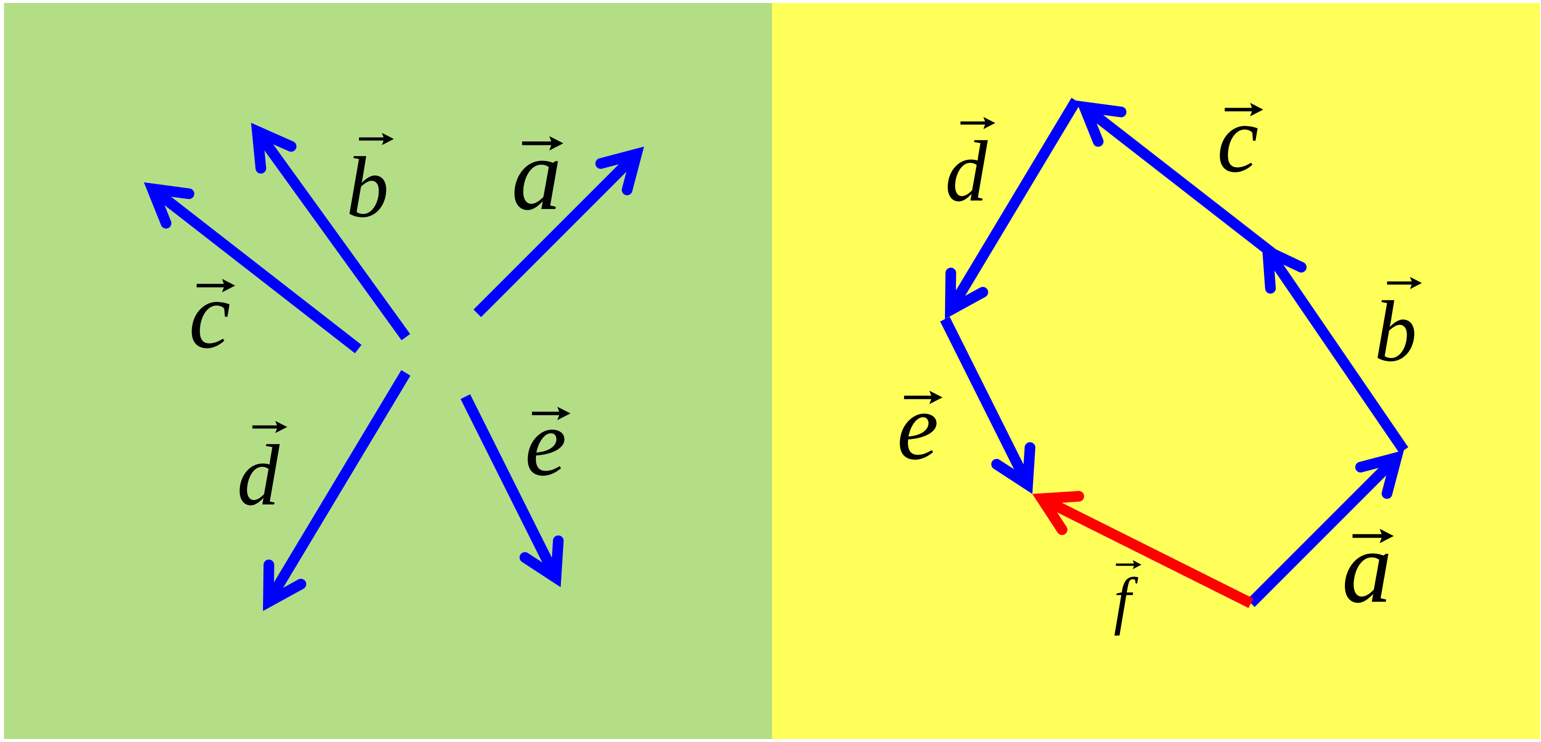


2. Vector addition is **associative-> (X+Y) + Z = X + (Y+Z)**



Vector Algebra

Head-to-tail Vektor Composition

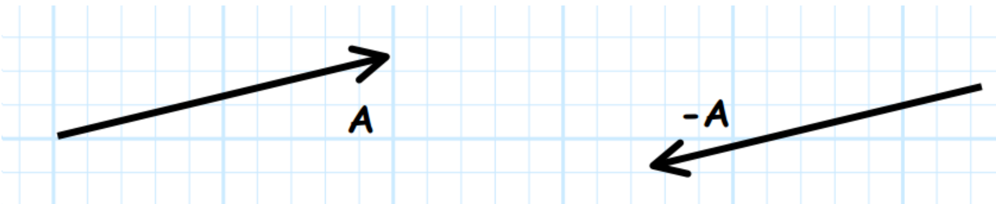


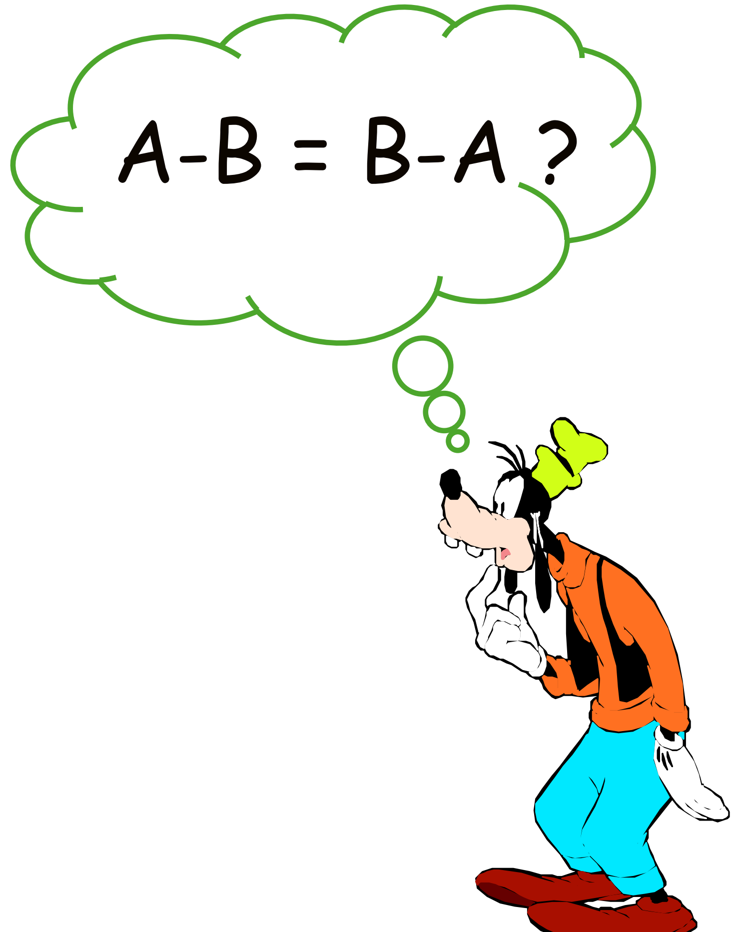
$$\vec{f} = \vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e}$$

Vector Algebra

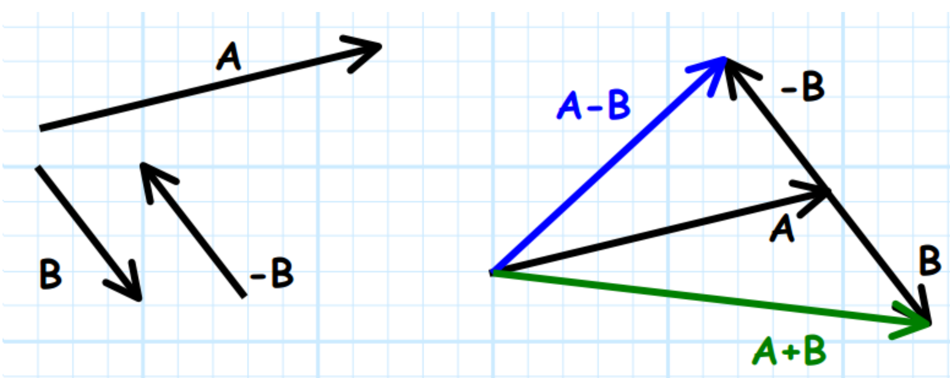
Vector Substraction

the negative of a vector is a vector with **equal magnitude** but **opposite direction.**





$$A + (-B) = A - B$$

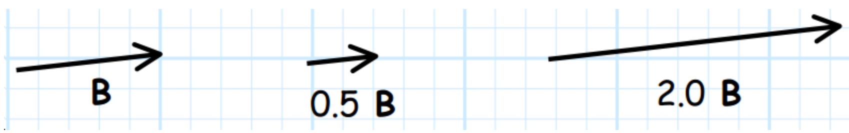


Vector Algebra

Scalar-Vector multiplication

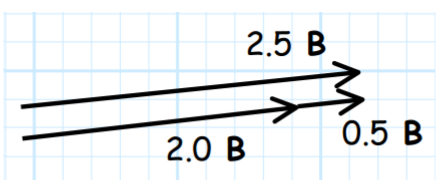
the product of a **scalar and a vector**—is a **vector!**

$$a\vec{B} = \vec{C} = a\left|\vec{B}\right|\hat{a}_B$$

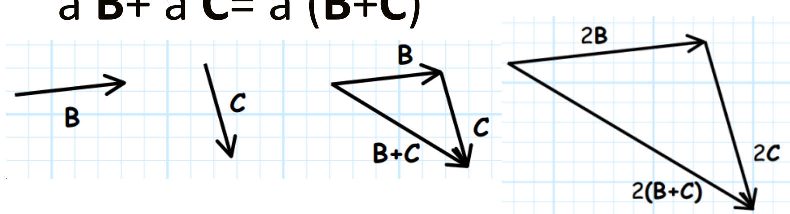


1. scalar-vector multiplication is **distributive**:

a **B**+ b **B** = (a+b) **B**



a **B**+ a **C**= a (**B**+**C**)

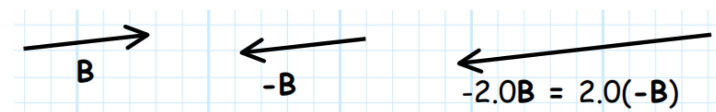


2. scalar-vector multiplication is **Commutative**:

a**B** = **B** a

3. Multiplication of a vector by a **negative scalar**:

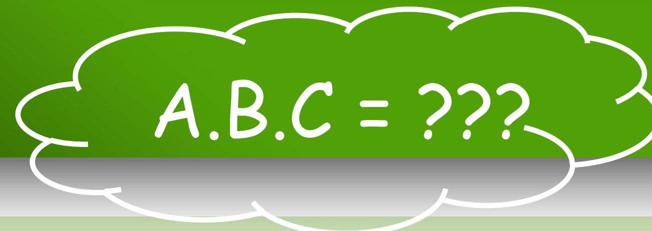
-a **B** = a(-**B**)

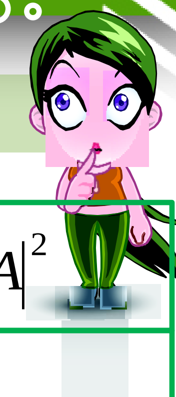


4. Division of a vector by a scalar is the same as multiplying the vector by the **inverse of the scalar**

$$\frac{\vec{B}}{a} = \left(\frac{1}{a}\right)\vec{B}$$

Vector Algebra

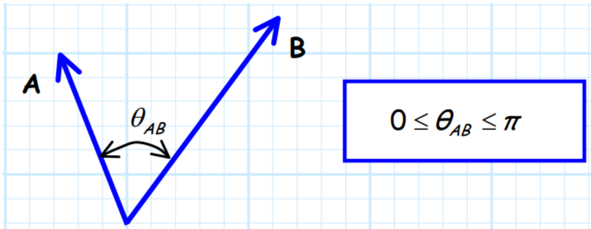




Dot Product / Perkalian Skalar

The dot product of two vectors is defined as:

$$\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos\theta_{AB}$$



$$0 \le \theta_{AB} \le \pi$$

IMPORTANT NOTE: The dot product is an operation involving **two vectors**, but the result is a **scalar**

1. the dot product is **commutative**

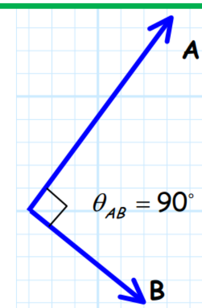
$$\vec{A}\bullet\vec{B} = \vec{B}\bullet\vec{A}$$

2. The dot product is **distributive** with addition

$$\vec{A}\bullet\left(\vec{B}+\vec{C}\right) = \vec{A}\bullet\vec{B} + \vec{A}\bullet\vec{C}$$

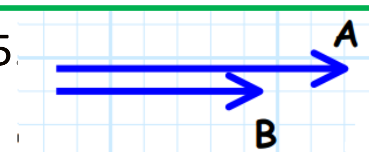
3. $\vec{A}\bullet\vec{A} = |A||A|\cos 0° = |A|^2$

4.



$$\vec{A}\bullet\vec{B} = 0$$

5.



$$\vec{A}\bullet\vec{B} = |\vec{A}||\vec{B}|\cos 0° = |\vec{A}||\vec{B}|$$

6.

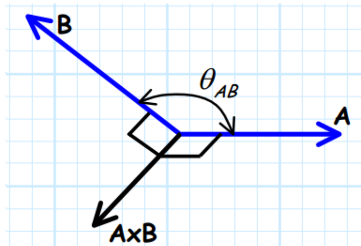
$$\vec{A}\bullet\vec{B} = |\vec{A}||\vec{B}|\cos 180° = -|\vec{A}||\vec{B}|$$

Vector Algebra

Cross Product

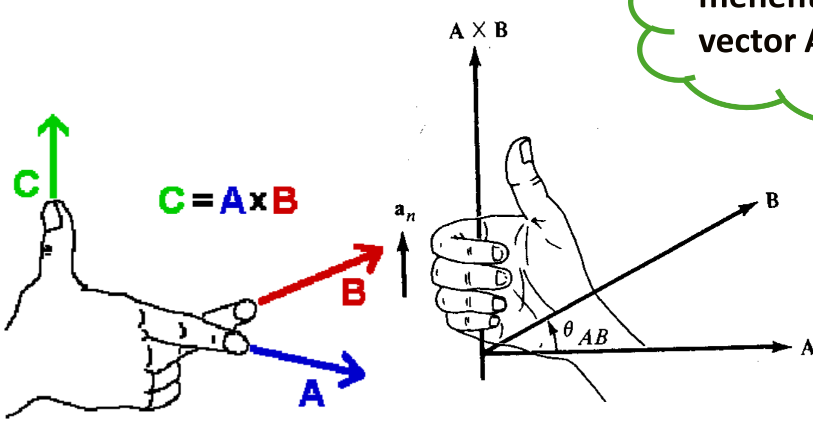
the product ofa **scalar and a vector**—is a **vector!**

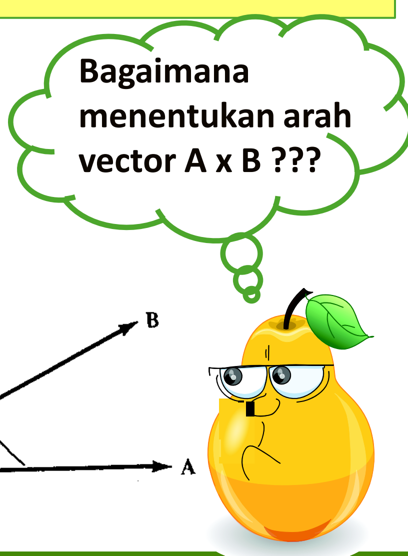
$$\mathbf{A}\times\mathbf{B} = \hat{a}_n |\mathbf{A}||\mathbf{B}|\sin\theta_{AB}$$

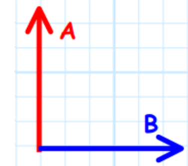


$$0 \le \theta_{AB} \le \pi$$

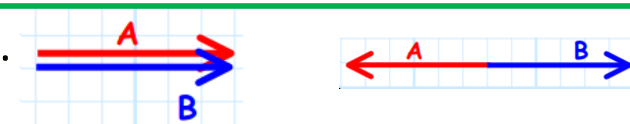
IMPORTANT NOTE: The cross product is an operation involving **two vectors**, and the result is also a **vector**





1. 

$$\vec{A}\times\vec{B} = \hat{a}_n |\vec{A}||\vec{B}|\sin 90^\circ = \hat{a}_n |\vec{A}||\vec{B}|$$

2. 

$$\vec{A}\times\vec{B} = \hat{a}_n |\vec{A}||\vec{B}|\sin 0^\circ = \hat{a}_n |\vec{A}||\vec{B}|\sin 180^\circ = 0$$

3. The cross product is **not commutative**

$$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$$

4. The cross product is also **not associative**

$$\left(\vec{A}\times\vec{B}\right)\times\vec{C} \neq \vec{A}\times\left(\vec{B}\times\vec{C}\right)$$

5. the cross product is **distributive**

$$\vec{A}\times\left(\vec{B}+\vec{C}\right) = \left(\vec{A}\times\vec{B}\right)+\left(\vec{A}\times\vec{C}\right)$$

Vector Algebra

Triple Product

The triple product is simply a **combination** of the **dot** and **cross products.**

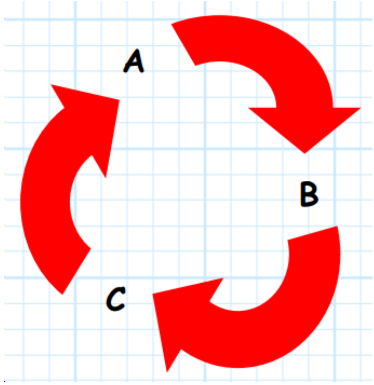
$$\vec{A}\bullet\vec{B}\times\vec{C}=\vec{A}\bullet\left(\vec{B}\times\vec{C}\right)$$

IMPORTANT NOTE: The triple product $\vec{A}\bullet\vec{B}\times\vec{C}$ results in a **scalar** value.

The Cyclic Property

$$\vec{A}\bullet\vec{B}\times\vec{C}=\vec{C}\bullet\vec{A}\times\vec{B}=\vec{B}\bullet\vec{C}\times\vec{A}$$

The cyclical rule means that the triple product is invariant to Shifts in the order of the vectors



Vector Algebra

Let's test your vector algebraic skills!
Can you evaluate the following expressions, and determine whether the result is a scalar (S), a vector(V), or neither (N) ??

IMPORTANT NOTE: If the expression initially results in a vector (or scalar), then after each manipulation, the result must also be a vector (or scalar)

1. $(\mathbf{A} \cdot \mathbf{B})\mathbf{C}$ ________
2. $\mathbf{A} + (\mathbf{B} \cdot \mathbf{C})$ ________
3. $\mathbf{A} \cdot (\mathbf{B} \cdot \mathbf{C})$ ________
4. $\mathbf{A}(\mathbf{B} \times \mathbf{C})$ ________
5. $\mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$ ________
6. $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) + \mathbf{C} \cdot (\mathbf{A} + \mathbf{B})$ ________
7. $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} \cdot \mathbf{D}$ ________

Intermezo

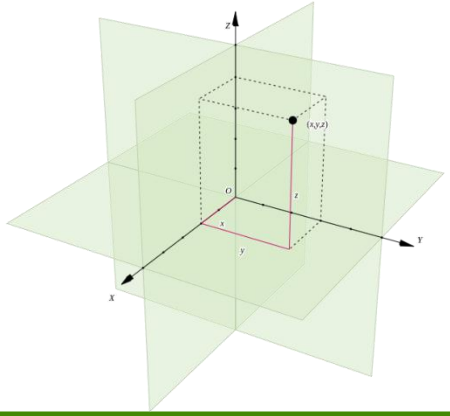
Find the baby

Coordinate Systems

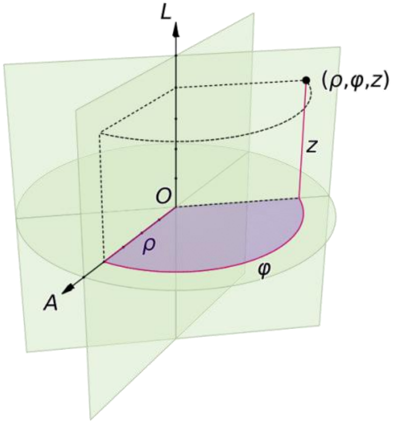
Coordinates system

- A set of 3 scalar values that define position and a set of 3 unit vectors that define direction form a **Coordinate system**
- The 3 scalar values used to define position are called **coordinates**
- All coordinates are defined with respect to an arbitrary point called the **origin**.
- The 3 unit vectors used to define direction are called **base vectors**

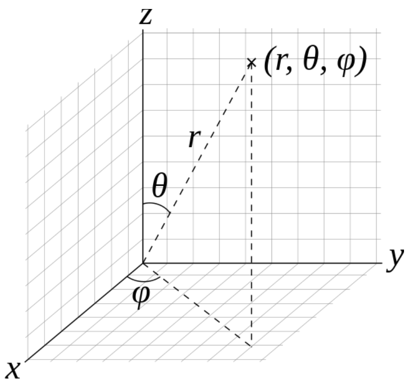
Cartesian Coordinate



Cylindrical Coordinate

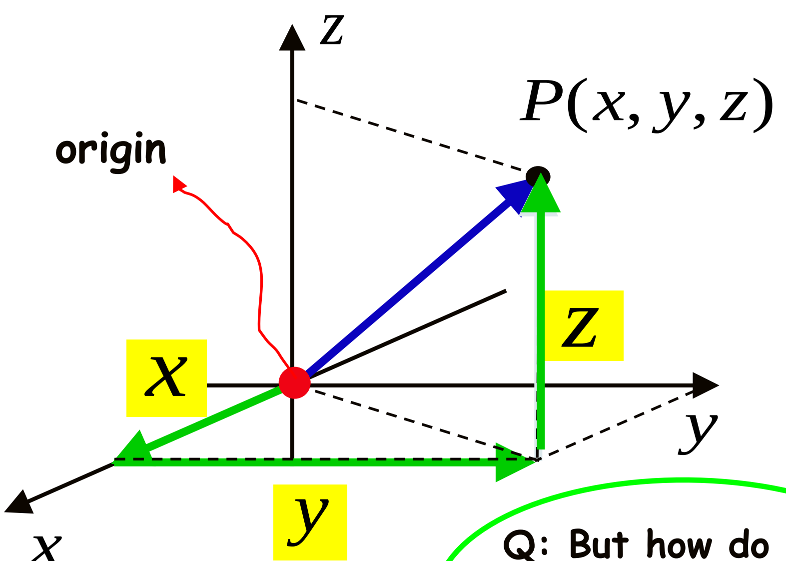


Spherical Coordinate



Coordinate Systems

Cartesian Coordinates system



- Kita dapat menentukan posisi (koordinat) **titik P** dengan <u>3 scalar values</u> yaitu x, y, dan z
- coordinate values in the Cartesian system effectively represent the distance from a plane intersecting the origin
  - to specify the direction of a vector quantity is by using a well defined <u>orthornormal</u> set of vectors known as **base vectors**. $\hat{a}_x,\ \hat{a}_y,\ \hat{a}_z$
  - Orthonormal :

    a. Each vector is a unit vector:

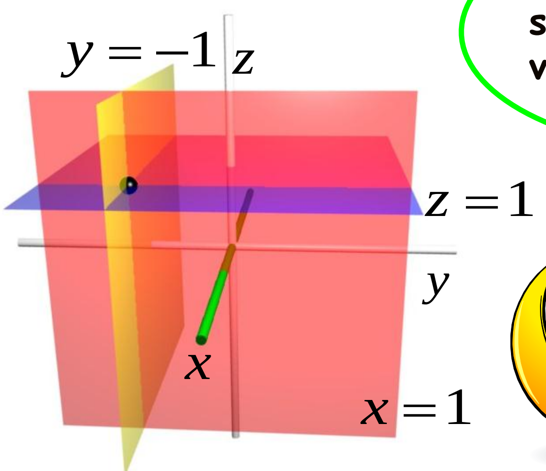
$$\hat{a}_x \bullet \hat{a}_x = \hat{a}_y \bullet \hat{a}_y = \hat{a}_z \bullet \hat{a}_z = 1$$

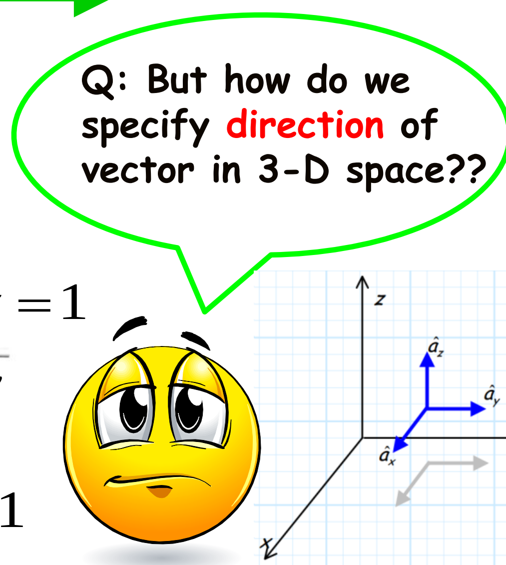
    a. Each vector is mutually orthogonal:

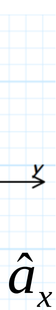
$$\hat{a}_x \bullet \hat{a}_y = \hat{a}_y \bullet \hat{a}_z = \hat{a}_x \bullet \hat{a}_z = 0$$

  - **Additionally**,a set of base vectors must be arranged such that:

$$\hat{a}_x \times \hat{a}_y = \hat{a}_z;\ \hat{a}_y \times \hat{a}_z = \hat{a}_x;\ \hat{a}_z \times \hat{a}_x = \hat{a}_y$$

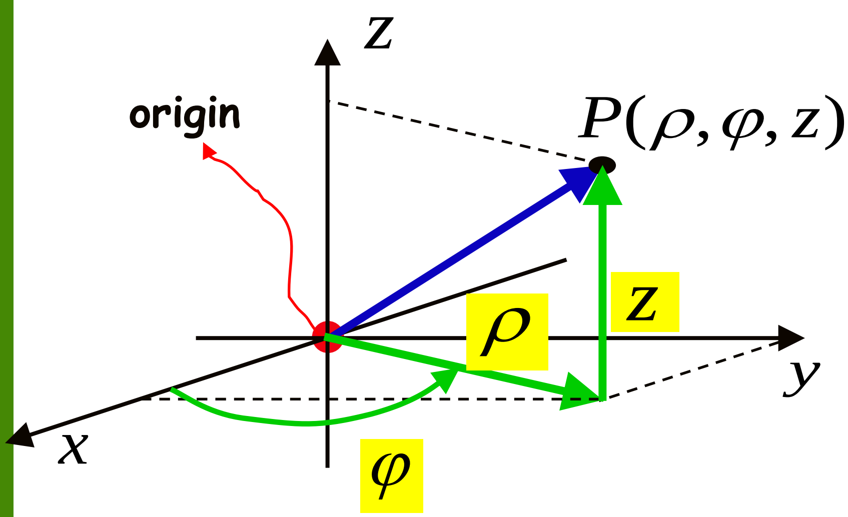




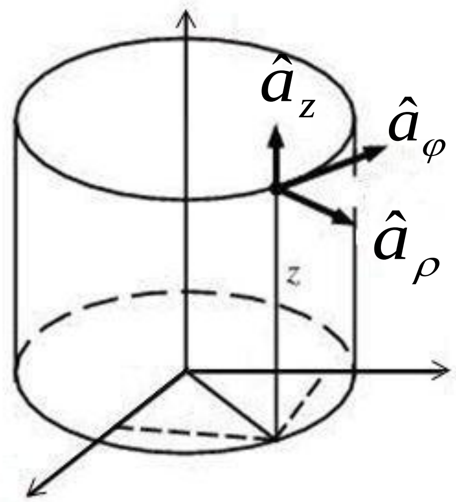


Coordinate Systems

Cylindrical Coordinates system



- Kita dapat menentukan posisi (koordinat) **titik P** pada koordinat tabung dengan 3 scalar values yaitu ρ, φ, z
- Keterangan :
  1) The value **ρ** indicates the distance of the point from the z-axis ($0 \le \rho < \infty$)
  2) The value **φ** indicates the rotation angle around the z-axis ($0 \le \varphi < 2\pi$)
  3) The value **z** indicates the distance of the point from the x-y (z = 0) plane ($-\infty < z < \infty$)

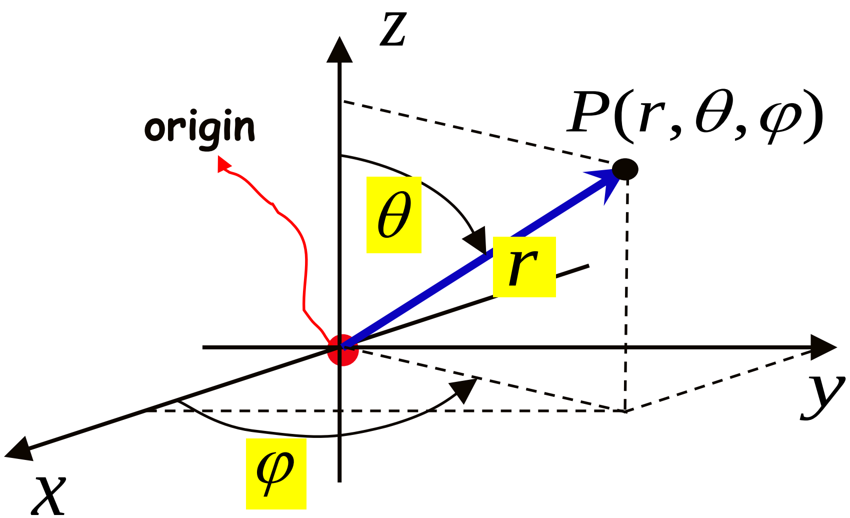


- to specify the direction of a vector quantity is by using a **base vectors** $\hat{a}_\rho \;\; \hat{a}_\varphi \;\; \hat{a}_z$
- Not like Cartesian base vectors, the cylindrical base vectors are **dependent** on position ($\hat{a}_\rho \;\; \hat{a}_\varphi$)
- set of base vectors must be arranged such that:

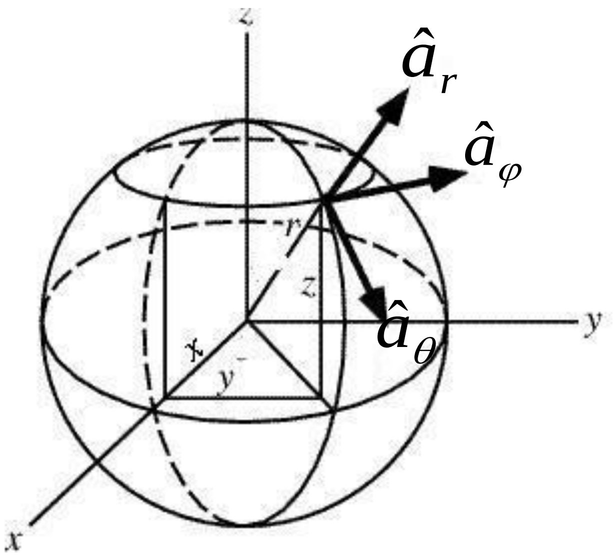
$$\hat{a}_\varphi \times \hat{a}_z = \hat{a}_\rho;\; \hat{a}_z \times \hat{a}_\rho = \hat{a}_\varphi;\; \hat{a}_\rho \times \hat{a}_\varphi = \hat{a}_z$$

Coordinate Systems

Spherical Coordinates system



- Kita dapat menentukan posisi (koordinat) **titik P** pada koordinat tabung dengan 3 scalar values yaitu r, θ, φ
- Keterangan :
  1) The value **r** ($0 \leq r < \infty$) expresses the distanceof the point from the origin.
  2) Angle **θ** ($0 \leq \theta \leq \pi$) represents the angle formed with the z-axis
  3) Angle **φ** ($0 \leq \varphi < 2\pi$) represents the rotation angle around the z-axis



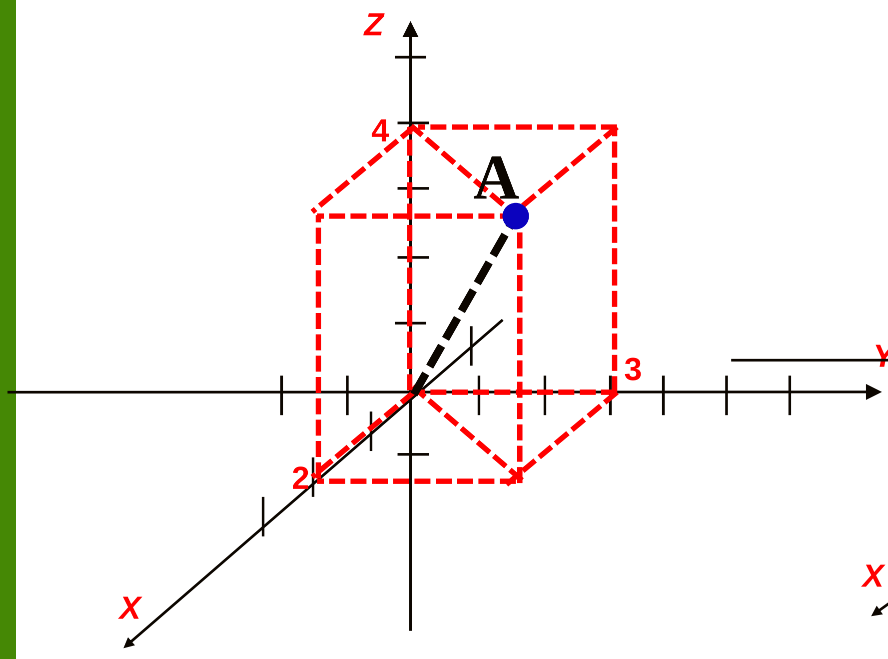
- to specify the direction of a vector quantity is by using a **base vectors** $\hat{a}_r \quad \hat{a}_\varphi \quad \hat{a}_\theta$
- the Spherical base vectors are **dependent** on position ($\hat{a}_r \quad \hat{a}_\varphi \quad \hat{a}_\theta$)
- set of base vectors must be arranged such that:

$$\hat{a}_\theta \times \hat{a}_\varphi = \hat{a}_r; \; \hat{a}_\varphi \times \hat{a}_r = \hat{a}_\theta; \; \hat{a}_r \times \hat{a}_\theta = \hat{a}_\varphi$$

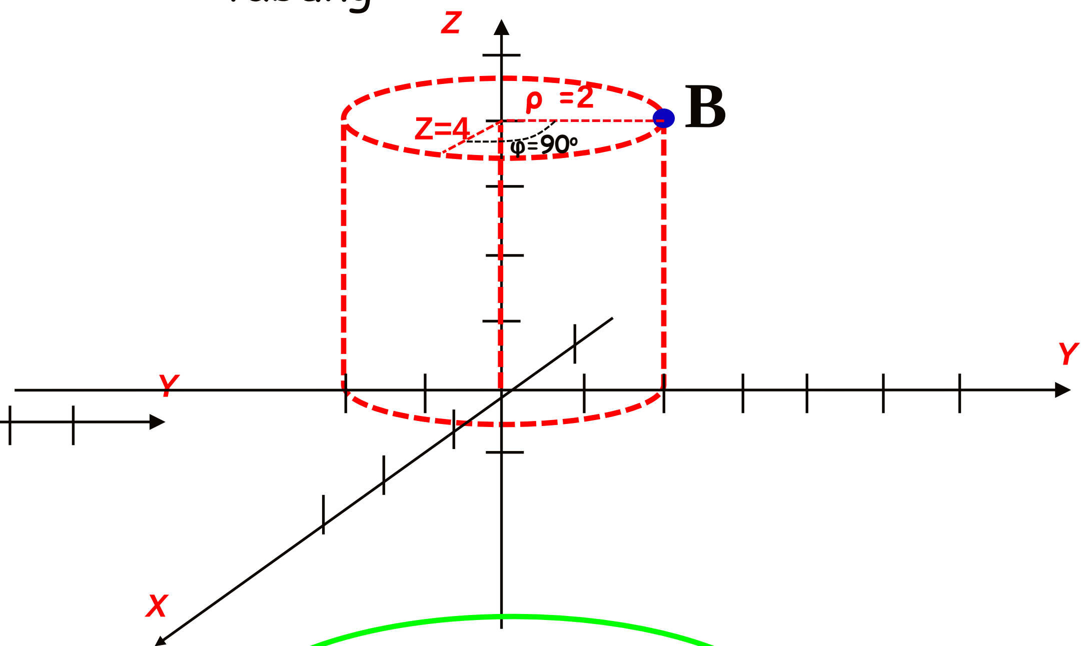
Coordinate Systems

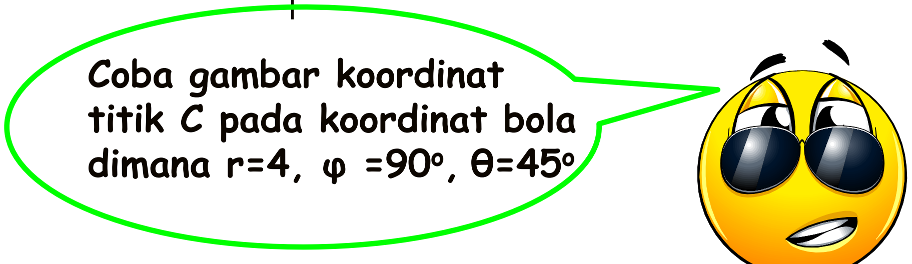
1. Posisi/coordinate suatu titik pada sistem koordinat

Titik **A** berposisi pada x=2, y=3, dan z=4 atau **A(2,3,4)** pada koordinat kertesian



Titik **B** berposisi pada **ρ** =2, **φ** =90°, dan z=4 atau **A(2,3,4)** pada koordinat tabung

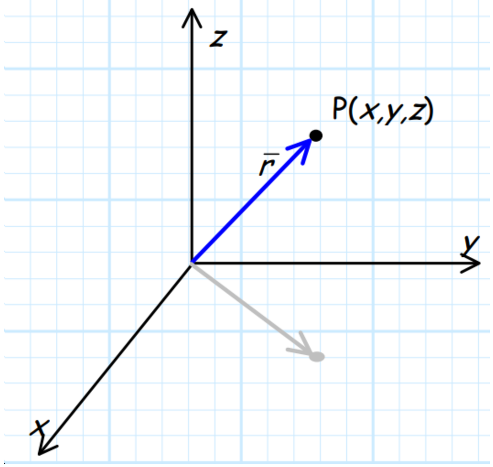




Coordinate Systems

2. Position Vector pada sistem koordinat

a vector beginning at the origin and extending outward to a point—is a very important and fundamental directed distance known as the **position vector** $\bar{r}$



Using the Cartesian coordinate system, the position vector can be explicitly written as:

$$\bar{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$$

Contoh : $\bar{r} = \hat{a}_x + 2\hat{a}_y - 3\hat{a}_z$
Sehingga dengan mudah kita bisa langsung menetukan koordinat titik yang dituju. X=1, y=2, z=-3

The magnitude of $\bar{r}$

$$|\bar{r}| = \sqrt{\bar{r}\cdot\bar{r}} = \sqrt{x^2 + y^2 + z^2} = r$$

Vektor satuan searah vektor $\bar{r}$

$$\hat{a}_r = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}{\sqrt{x^2 + y^2 + z^2}}$$

NEVER, EVER express the position vector in either of Cylindrical base vector or Spherical base vector

$$\bar{r} \neq \rho\,\hat{a}_\rho + \phi\,\hat{a}_\phi + z\,\hat{a}_z$$

$$\bar{r} \neq r\,\hat{a}_r + \theta\,\hat{a}_\theta + \phi\,\hat{a}_\phi$$

Coordinate Systems

Latihan Soal

Titik A(3,4,5) terletak dalam koordinat Cartesius. Tentukan :

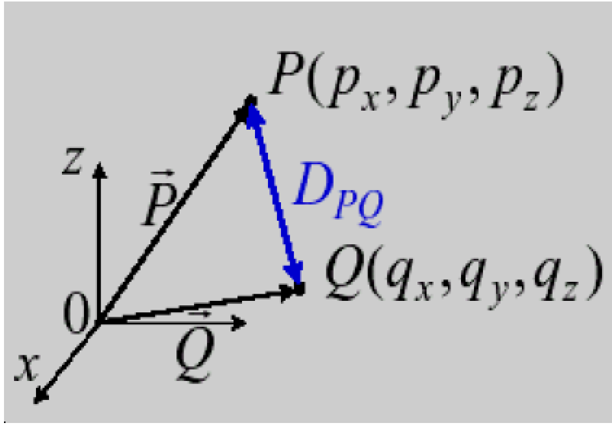
a) Gambar vektor posisi A
b) Tulis vektor posisi A
c) Hitung besar/magnitude vektor A
d) Tentukan vektor satuan searah A

Jarak antar 2 titik (koordinat)

- Position Vector berguna saat kita ingin menentukan jarak antar 2 titik dalam suatu sistem koordinat
- Misalkan dua buah titik **P** dan **Q**

$$\vec{P} = p_x\hat{a}_x + p_y\hat{a}_y + p_z\hat{a}_z$$

$$\vec{Q} = q_x\hat{a}_x + q_y\hat{a}_y + q_z\hat{a}_z$$



- Jarak antara dua titik P dan Q adalah magnitudo dari perbedaan vektor **P** dan **Q**

$$\overrightarrow{PQ} = (q_x - p_x)\hat{a}_x + (q_y - p_y)\hat{a}_y + (q_z - p_z)\hat{a}_z$$

- Jarak titik P dan Q

$$D_{PQ} = \sqrt{(q_x - p_x)^2 + (q_y - p_y)^2 + (q_z - p_z)^2}$$

Coordinate Systems

3. Scalar Field pada sistem koordinat

- a scalar field is a scalar quantity that is a function of position and/or time
- Scalar field can be written as :

$$A(x,y,z) = f(x,y,z)$$

$$A(\rho,\varphi,z) = f(\rho,\varphi,z)$$

$$A(r,\theta,\varphi) = f(r,\theta,\varphi)$$

Contoh

$$g(x,y,z) = (x^2 + y^2)yz$$

Coba Anda Gambarkan scalar field disamping pada koordinat kartesian

Coordinate Systems

4. Vector pada sistem koordinat

any vector can be written as a sum of <u>three vectors components</u>!

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

$$\vec{A} = A_\rho\hat{a}_\rho + A_\varphi\hat{a}_\varphi + A_z\hat{a}_z$$

$$\vec{A} = A_r\hat{a}_r + A_\varphi\hat{a}_\varphi + A_\theta\hat{a}_\theta$$

- Each of these three vectors point in one of the three orthogonal directions
- The magnitude of the vector is determined by the scalar values Ax, Ay, and Az that called the **<u>scalar components</u>** <u>of vector A.</u>
- The vectors $A_x\hat{a}_x, A_y\hat{a}_y, A_z\hat{a}_z$ are called the **<u>vector components of A.</u>**

- Menentukan scalar component :

$$\vec{A}\bullet\hat{a}_x = \left(A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z\right)\bullet\hat{a}_x$$

$$= A_x\hat{a}_x\bullet\hat{a}_x + A_y\hat{a}_y\bullet\hat{a}_x + A_z\hat{a}_z\bullet\hat{a}_x = A_x$$

$$A_x = \vec{A}\bullet\hat{a}_x \quad \Rightarrow \quad A_x = \vec{A}\bullet\hat{a}_x, \quad A_y = \vec{A}\bullet\hat{a}_y, \quad A_z = \vec{A}\bullet\hat{a}_z$$

- Sehingga :

$$\vec{A} = (\vec{A}\bullet\hat{a}_x)\hat{a}_x + (\vec{A}\bullet\hat{a}_y)\hat{a}_y + (\vec{A}\bullet\hat{a}_z)\hat{a}_z$$

- Menentukan Magnitude Vector:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{A_\rho^2 + A_\varphi^2 + A_z^2}$$

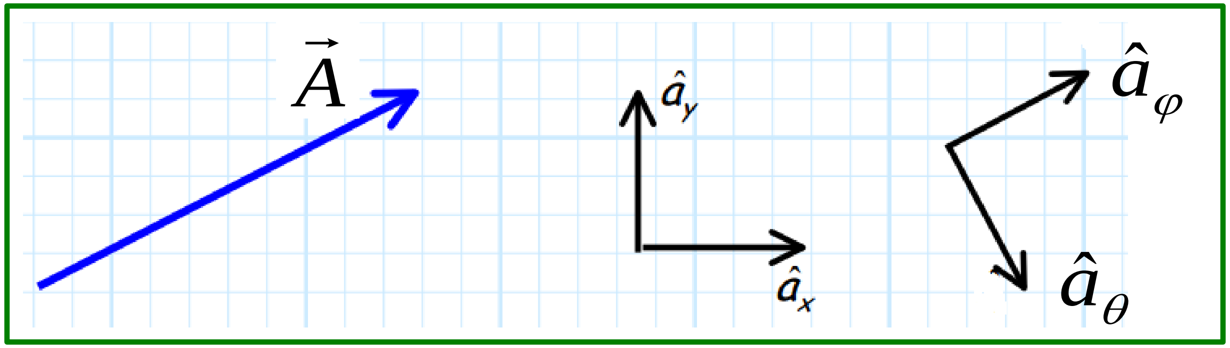
$$= \sqrt{A_r^2 + A_\theta^2 + A_\varphi^2}$$

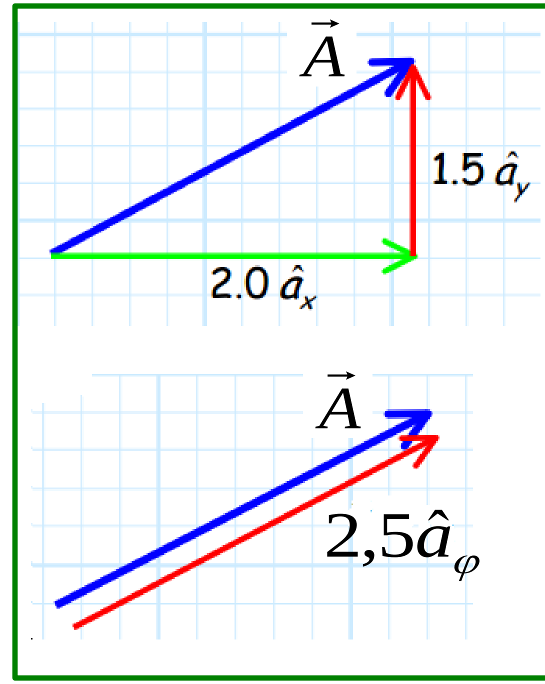
- Menentukan vektor satuan searah vektor $\bar{A}$

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Coordinate Systems

Contoh





$$A_x = \vec{A} \bullet \hat{a}_x = 2 \qquad A_r = \vec{A} \bullet \hat{a}_r = 0$$

$$A_y = \vec{A} \bullet \hat{a}_y = 1,5 \qquad A_\varphi = \vec{A} \bullet \hat{a}_\varphi = 2,5$$

$$A_z = \vec{A} \bullet \hat{a}_z = 0 \qquad A_\theta = \vec{A} \bullet \hat{a}_\theta = 0$$

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$
$$= 2\hat{a}_x + 1,5\hat{a}_y$$

$$\vec{A} = 2,5\hat{a}_\varphi$$

$$\left|\vec{A}\right| = \sqrt{A_x^{\ 2} + A_y^{\ 2} + A_z^{\ 2}}$$
$$= \sqrt{2^2 + 1,5^2} = \sqrt{6,25} = 2,5$$
$$= \sqrt{A_r^{\ 2} + A_\theta^{\ 2} + A_\varphi^{\ 2}}$$
$$= \sqrt{2,5^2} = 2,5$$

$$\hat{a}_A = \frac{\vec{A}}{\left|\vec{A}\right|} = \frac{2\hat{a}_x + 1,5\hat{a}_y}{2,5} = 0,8\hat{a}_x + 0,6\hat{a}_y$$

$$\hat{a}_A = \frac{\vec{A}}{\left|\vec{A}\right|} = \frac{2,5\hat{a}_\varphi}{2,5} = \hat{a}_\varphi$$

Coordinate Systems

Latihan soal

1. Gambarkan vektor berikut dalam sistem koordinat silinder
   a) $A = 3\mathbf{a}_\rho + 2\mathbf{a}_\phi + \mathbf{a}_z$ berpangkal di M(2,0,0)
   b) $B = 3\mathbf{a}_\rho + 2\mathbf{a}_\phi + \mathbf{a}_z$ berpangkal di N(2,π/2,0)
2. Gambarkan vektor berikut pada sistem koordinat bola
   a) $C = 3\mathbf{a}_r + \mathbf{a}_\theta + 2\mathbf{a}_\phi$ berpangkal di M(2, π/2, 0)
   b) $D = 3\mathbf{a}_r + \mathbf{a}_\theta + 2\mathbf{a}_\phi$ berpangkal di N(2, π/2, π/2)

Coordinate Systems

5. Vector Fields pada sistem koordinat

- a vector field is a vector quantity that is a function of position and/or time
- When we express a vector field using orthonormal base vectors, the scalar component of each direction is a scalar field—**a scalar function of position!**

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$
$$\vec{A} = A_\rho\hat{a}_\rho + A_\varphi\hat{a}_\varphi + A_z\hat{a}_z$$
$$\vec{A} = A_r\hat{a}_r + A_\varphi\hat{a}_\varphi + A_\theta\hat{a}_\theta$$

$$\vec{A}(x,y,z) = A_x(x,y,z)\hat{a}_x + A_y(x,y,z)\hat{a}_y + A_z(x,y,z)\hat{a}_z$$
$$\vec{A}(\rho,\varphi,z) = A_\rho(\rho,\varphi,z)\hat{a}_\rho + A_\varphi(\rho,\varphi,z)\hat{a}_\varphi + A_z(\rho,\varphi,z)\hat{a}_z$$
$$\vec{A}(r,\varphi,\theta) = A_r(r,\varphi,\theta)\hat{a}_r + A_\varphi(r,\varphi,\theta)\hat{a}_\varphi + A_\theta(r,\varphi,\theta)\hat{a}_\theta$$

Contoh

$$\vec{A}(x,y,z) = (x^2+y^2)\hat{a}_x + \frac{xz}{y}\hat{a}_y + (3-y)\hat{a}_z$$

- Often, vector fields must be written as variables of three spatial coordinates, as well as a time variable t and known as a **dynamic vector field**.

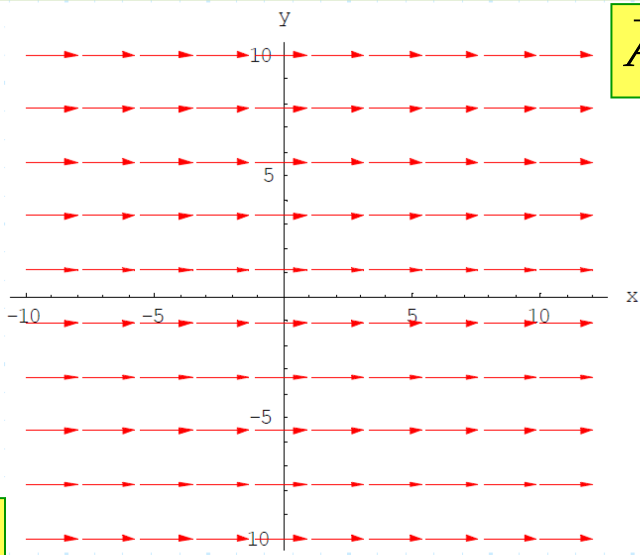
Contoh

$$\vec{A}(x,y,z,t) = (x^2+y^2)t\hat{a}_x + \frac{xz}{y}t^2\hat{a}_y + (3-y+4t)\hat{a}_z$$

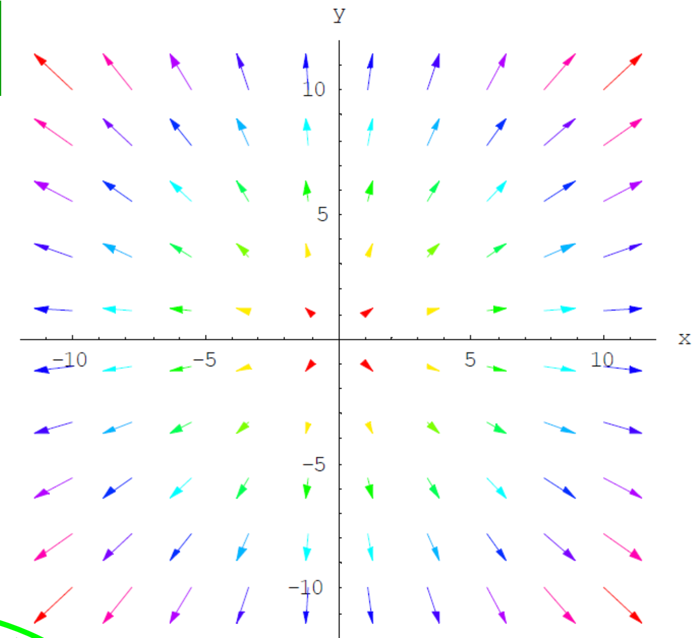
Coordinate Systems

Gallery of Vector Scalar and Vector Field

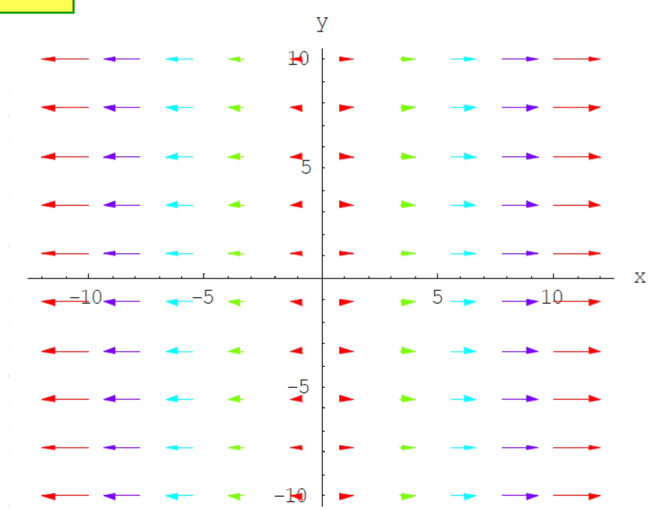
$\vec{A} = \hat{a}_x$



$\vec{A} = x\hat{a}_x + y\hat{a}_y$



$\vec{A} = x\hat{a}_x$



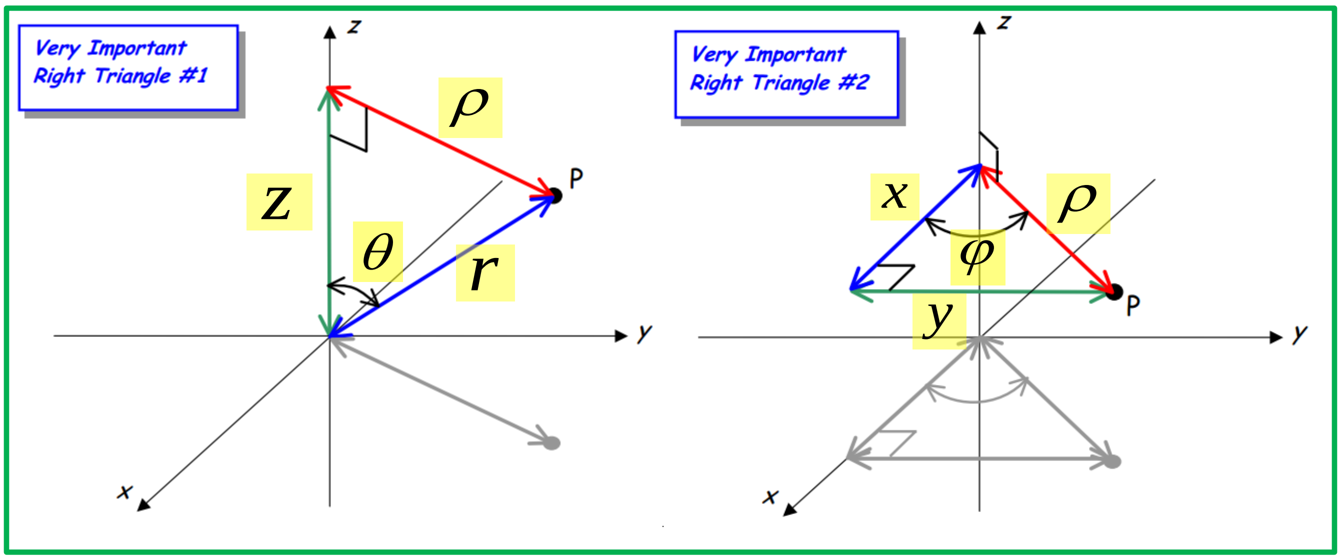
Coba gambarkan vector field beikut :

$$\vec{A} = \hat{a}_\rho$$

Coordinate Systems

Point Location/Coordinate and scalar field Transformation

Transformation of expressing point location/Coordinate and scalar field in terms of cartesian coordinate to cylindrical or spherical coordinates or verse versa



Cylindrical ke Spherical

$$(\rho, \varphi, z) \Rightarrow (r, \theta, \varphi)$$

$$r = \sqrt{\rho^2 + z^2}$$

$$\theta = \tan^{-1}\left(\frac{\rho}{z}\right)$$

$$\varphi = \varphi$$

Spherical ke Cylindrical

$$(r, \theta, \varphi) \Rightarrow (\rho, \varphi, z)$$

$$\rho = r \sin\theta$$

$$\varphi = \varphi$$

$$z = r\cos\theta$$

Cartesian ke Cylindrical

$$(x, y, z) \Rightarrow (\rho, \varphi, z)$$

$$\rho = \sqrt{x^2 + y^2}$$

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$z = z$$

Cylindrical ke Cartesian

$$(\rho, \varphi, z) \Rightarrow (x, y, z)$$

$$x = \rho \cdot \cos\varphi$$

$$y = \rho \cdot \sin\varphi$$

$$z = z$$

Cartesian ke Spherical

$$(x, y, z) \Rightarrow (r, \theta, \varphi)$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$$

Spherical ke Cartesian

$$(r, \theta, \varphi) \Rightarrow (x, y, z)$$

$$x = r \cdot \sin\theta \cdot \cos\varphi$$

$$y = r \cdot \sin\theta \cdot \sin\varphi$$

$$z = r \cdot \cos\theta$$

Coordinate Systems

Latihan Soal

1. Titik A(-3,-3,2) terletak dalam koordinat Cartesius. Tentukan :
   a. Tentukan koordinat titik tersebut pada koordinat tabung
   b. Tentukan koordinat titik tersebut pada koordinat bola

2. Suatu scalar field $g(\rho,\varphi,z)=\rho^3 z\sin\varphi$
   tulis fungsi g di atas dengan expresi koordinat kartesian

Coordinate Systems

Vector Transformation

Transformation of expressing Vector field in terms of cartesian coordinate to cylindrical or spherical coordinates or verse versa

$\bullet$	$\vec{a}_\rho$	$\vec{a}_\phi$	$\vec{a}_z$
$\vec{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\vec{a}_y$	$\sin\phi$	$\cos\phi$	0
$\vec{a}_z$	0	0	1

Vektor Field di Koordinat Cartesian

$$\mathbf{A}(x,y,z) = A_x(x,y,z)\,\hat{a}_x + A_y(x,y,z)\,\hat{a}_y + A_z(x,y,z)\,\hat{a}_z$$

Ingat!!!

$$\mathbf{A} = (\mathbf{A}\cdot\hat{a}_x)\hat{a}_x + (\mathbf{A}\cdot\hat{a}_y)\hat{a}_y + (\mathbf{A}\cdot\hat{a}_z)\hat{a}_z$$

.	$\vec{a}_r$	$\vec{a}_\theta$	$\vec{a}_\phi$
$\vec{a}_x$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\vec{a}_y$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\vec{a}_z$	$\cos\theta$	$-\sin\theta$	0

Vektor Field di Koordinat Silinder

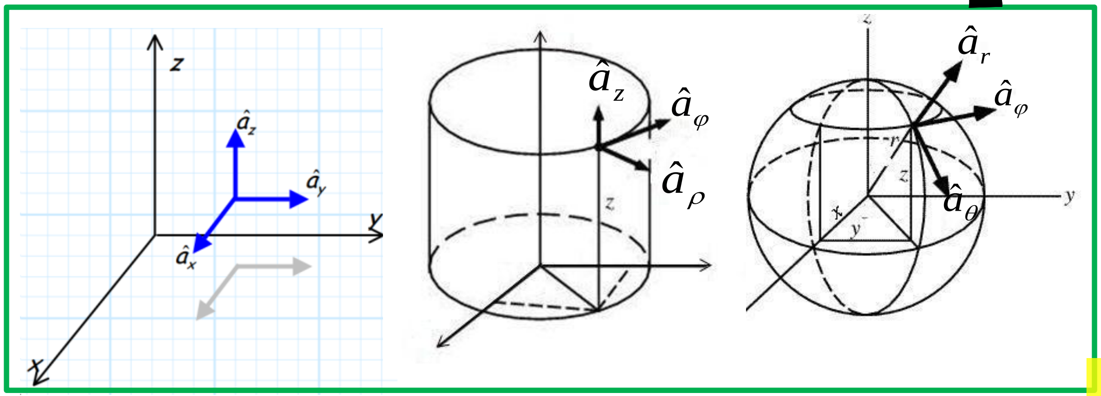
$$\vec{A}(\rho,\varphi,z) = A_\rho(\rho,\varphi,z)\hat{a}_\rho + A_\varphi(\rho,\varphi,z)\hat{a}_\varphi + A_z(\rho,\varphi,z)\hat{a}_z$$
$$= \left(\vec{A}(x,y,z)\bullet\hat{a}_\rho\right)\hat{a}_\rho + \left(\vec{A}(x,y,z)\bullet\hat{a}_\varphi\right)\hat{a}_\varphi + \left(\vec{A}(x,y,z)\bullet\hat{a}_z\right)\hat{a}_z$$

Vektor Field di Koordinat Bola

$$\vec{A}(r,\theta,\varphi) = A_r(r,\theta,\varphi)\hat{a}_r + A_\theta(r,\theta,\varphi)\hat{a}_\theta + A_\varphi(r,\theta,\varphi)\hat{a}_\varphi$$
$$= \left(\vec{A}(x,y,z)\bullet\hat{a}_r\right)\hat{a}_r + \left(\vec{A}(x,y,z)\bullet\hat{a}_\theta\right)\hat{a}_\theta + \left(\vec{A}(x,y,z)\bullet\hat{a}_\varphi\right)\hat{a}_\varphi$$

Coordinate Systems

Vector Transformation



$$\vec{A}(x,y,z) = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

$$\vec{A}(\rho,\varphi,z) = A_\rho\hat{a}_\rho + A_\varphi\hat{a}_\varphi + A_z\hat{a}_z$$

$$\vec{A}(r,\theta,\varphi) = A_r\hat{a}_r + A_\theta\hat{a}_\theta + A_\varphi\hat{a}_{\varphi\,z}$$

Cartesian ke Spherical $(x,y,z) \Rightarrow (r,\theta,\varphi)$

$$A_r = A_x\cos\varphi\sin\theta + A_y\sin\varphi\sin\theta + A_z\cos\theta$$

$$A_\theta = A_x\cos\varphi\cos\theta + A_y\sin\varphi\cos\theta - A_z\sin\theta$$

$$A_\varphi = -A_x\sin\varphi + A_y\cos\varphi$$

Spherical ke Cartesian $(r,\theta,\varphi) \Rightarrow (x,y,z)$

$$A_x = A_r\sin\theta\cos\varphi + A_\theta\cos\theta\cos\varphi - A_\varphi\sin\varphi$$

$$A_y = A_r\sin\theta\sin\varphi + A_\theta\cos\theta\sin\varphi + A_\varphi\cos\varphi$$

$$A_z = A_r\cos\theta - A_\theta\sin\theta$$

Cylindrical ke Spherical $(\rho,\varphi,z) \Rightarrow (r,\theta,\varphi)$

$$A_r = A_\rho\sin\theta + A_z\cos\theta$$

$$A_\theta = A_\rho\cos\theta - A_z\sin\theta$$

$$A_\varphi = A_\varphi$$

Cartesian ke Cylindrical $(x,y,z) \Rightarrow (\rho,\varphi,z)$

$$A_\rho = A_x\cos\varphi + A_y\sin\varphi$$

$$A_\varphi = -A_x\sin\varphi + A_y\cos\varphi$$

$$A_z = A_z$$

Cylindrical ke Cartesian $(\rho,\varphi,z) \Rightarrow (x,y,z)$

$$A_x = A_\rho\cos\varphi - A_\varphi\sin\varphi$$

$$A_y = A_\rho\sin\varphi + A_\varphi\cos\varphi$$

$$A_z = A_z$$

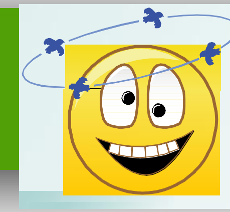
Spherical ke Cylindrical $(r,\theta,\varphi) \Rightarrow (\rho,\varphi,z)$

$$A_\rho = A_r\sin\theta + A_\theta\cos\theta$$

$$A_\varphi = A_\varphi$$

$$A_z = A_r\cos\theta - A_\theta\sin\theta$$

Coordinate Systems



Latihan soal

1. Vektor $\bar{A} = 3\hat{a}_\rho + 4\hat{a}_\varphi + 5\hat{a}_z$ berada pada sistem koordinat silinder dengan titik pangkal di $(10,\pi/2,0)$. Tentukan penulisan vektor ini pada sistem koordinat kartesian!
2. Suatu vektor $\vec{B}(x,y,z) = y\hat{a}_x - x\hat{a}_y + z\hat{a}_z$ tentukan:
   a. Expresi vektor tersebut pada koordinat tabung!
   b. Expresi vektor tersebut pada koordinat bola!

Coordinate Systems

Vektor Algebra pada coordinate systems

Misalnya kita memiliki dua buah vektor pada koordinat kartesian :

$$\mathbf{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$$

$$\mathbf{B} = B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z$$

Addition and Subtraction

$$\begin{aligned}\mathbf{A}+\mathbf{B} &= \left(A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z\right)+\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &= A_x\,\hat{a}_x + B_x\,\hat{a}_x + A_y\,\hat{a}_y + B_y\,\hat{a}_y + A_z\,\hat{a}_z + B_z\,\hat{a}_z\\ &= \left(A_x + B_x\right)\hat{a}_x + \left(A_y + B_y\right)\hat{a}_y + \left(A_z + B_z\right)\hat{a}_z\end{aligned}$$

$$\begin{aligned}\mathbf{A}-\mathbf{B} &= \left(A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z\right)-\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &= A_x\,\hat{a}_x - B_x\,\hat{a}_x + A_y\,\hat{a}_y - B_y\,\hat{a}_y + A_z\,\hat{a}_z - B_z\,\hat{a}_z\\ &= \left(A_x - B_x\right)\hat{a}_x + \left(A_y - B_y\right)\hat{a}_y + \left(A_z - B_z\right)\hat{a}_z\end{aligned}$$

Vektor-scalar multiplication

$$\begin{aligned}a\mathbf{B} &= a\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &= aB_x\,\hat{a}_x + aB_y\,\hat{a}_y + aB_z\,\hat{a}_z\\ &= \left(aB_x\right)\hat{a}_x + \left(aB_y\right)\hat{a}_y + \left(aB_z\right)\hat{a}_z\end{aligned}$$

Dot Product

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= \left(A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z\right)\cdot\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &= A_x\,\hat{a}_x\cdot\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &\quad + A_y\,\hat{a}_y\cdot\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &\quad + A_z\,\hat{a}_z\cdot\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &= A_x\,B_x\left(\hat{a}_x\cdot\hat{a}_x\right) + A_x\,B_y\left(\hat{a}_x\cdot\hat{a}_y\right) + A_x\,B_z\left(\hat{a}_x\cdot\hat{a}_z\right)\\ &\quad + A_y\,B_x\left(\hat{a}_y\cdot\hat{a}_x\right) + A_y\,B_y\left(\hat{a}_y\cdot\hat{a}_y\right) + A_y\,B_y\left(\hat{a}_y\cdot\hat{a}_z\right)\\ &\quad + A_z\,B_x\left(\hat{a}_z\cdot\hat{a}_x\right) + A_z\,B_y\left(\hat{a}_z\cdot\hat{a}_y\right) + A_z\,B_z\left(\hat{a}_z\cdot\hat{a}_z\right) = A_xB_x + A_yB_y + A_zB_z\end{aligned}$$

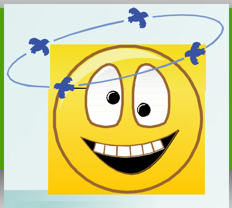
Cross Product

$$\begin{aligned}\mathbf{A}\times\mathbf{B} &= \left(A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z\right)\times\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &= A_x\,\hat{a}_x\times\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &\quad + A_y\,\hat{a}_y\times\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &\quad + A_z\,\hat{a}_z\times\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &= A_xB_x\left(\hat{a}_x\times\hat{a}_x\right) + A_xB_y\left(\hat{a}_x\times\hat{a}_y\right) + A_xB_z\left(\hat{a}_x\times\hat{a}_z\right)\\ &\quad + A_yB_x\left(\hat{a}_y\times\hat{a}_x\right) + A_yB_y\left(\hat{a}_y\times\hat{a}_y\right) + A_yB_z\left(\hat{a}_y\times\hat{a}_z\right)\\ &\quad + A_zB_x\left(\hat{a}_z\times\hat{a}_x\right) + A_zB_y\left(\hat{a}_z\times\hat{a}_y\right) + A_zB_z\left(\hat{a}_z\times\hat{a}_z\right)\end{aligned}$$

$$\vec{A}\times\vec{B} = \begin{bmatrix} \overset{\oplus}{\hat{a}_x} & \overset{\oplus}{\hat{a}_y} & \overset{\oplus}{\hat{a}_z} \\ A_x & A_y & A_z \\ \underset{-}{B_x} & \underset{-}{B_y} & \underset{-}{B_z} \end{bmatrix} \begin{matrix} \hat{a}_x & \hat{a}_y \\ A_x & A_y \\ B_x & B_y \end{matrix}$$

$$\mathbf{A}\times\mathbf{B} = \left(A_yB_z - A_zB_y\right)\hat{a}_x + \left(A_zB_x - A_xB_z\right)\hat{a}_y + \left(A_xB_y - A_yB_x\right)\hat{a}_z$$

Coordinate Systems



Latihan soal

1. Diberikan tiga vektor pada sistem koordinat Kartesian dibawah ini :

$$\vec{A} = \hat{a}_x + \hat{a}_y$$

$$\vec{B} = \hat{a}_x + 2\hat{a}_y - 2\hat{a}_z$$

$$\vec{C} = \hat{a}_y + 2\hat{a}_z$$

tentukan hasil dari operasi-operasi vektor dibawah ini :

$a)\quad \vec{A} + \vec{B}$

$b)\quad \vec{B} - \vec{C}$

$c)\quad 4\vec{C}$

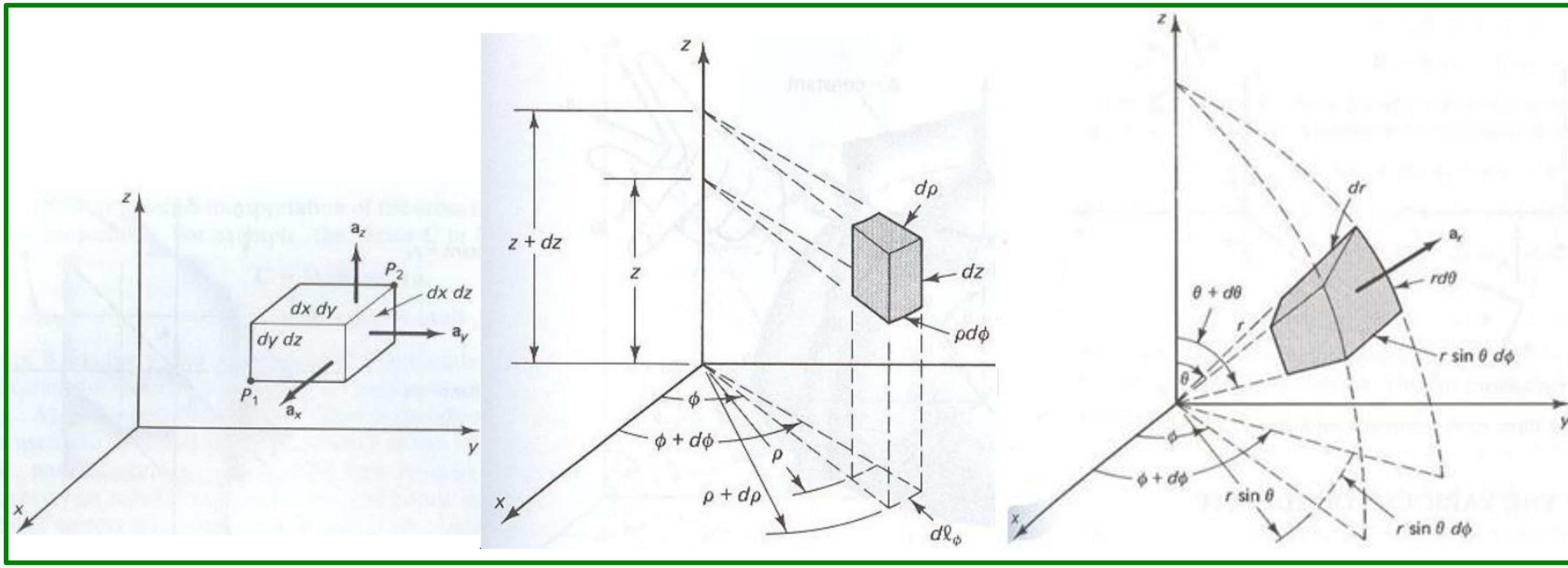
$d)\quad \vec{A} \bullet \vec{B}$

$e)\quad \vec{A} \times \vec{B}$

$f)\quad \vec{A} \bullet \vec{B} \times \vec{C}$

Coordinate Systems

Elemen Perpindahan, Elemen Luas, dan Elemen Volume

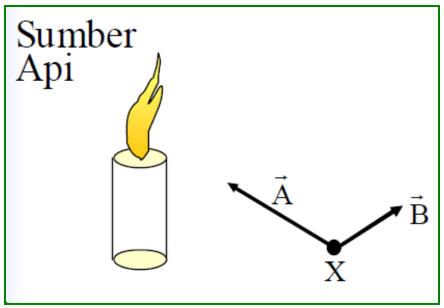


Elemen perpindahan $d\vec{L} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$	**Elemen perpindahan** $d\vec{L} = d\rho\,\hat{a}_\rho + \rho d\varphi\,\hat{a}_\phi + dz\,\hat{a}_z$	**Elemen perpindahan** $d\vec{L} = dr\,\hat{a}_r + rd\theta\,\hat{a}_\theta + r\sin\theta d\varphi\,\hat{a}_\varphi$
Elemen luas $d\vec{S}_x = dydz\,\vec{a}_x$ $d\vec{S}_y = dxdz\,\vec{a}_y$ $d\vec{S}_z = dxdy\,\vec{a}_z$	**Elemen luas** $d\vec{S}_\rho = \rho d\varphi dz\,\hat{a}_\rho$ $d\vec{S}_\varphi = d\rho dz\,\hat{a}_\varphi$ $d\vec{S}_z = \rho d\rho d\varphi\,\hat{a}_z$	**Elemen luas** $d\vec{S}_r = r\sin\theta d\theta d\varphi\,\hat{a}_r$ $d\vec{S}_\theta = r\sin\theta drd\varphi\,\hat{a}_\theta$ $d\vec{S}_\varphi = rdrd\theta\,\hat{a}_\varphi$
Elemen volume $dV = dxdydz$	**Elemen volume** $dV = \rho d\rho d\varphi dz$	**Elemen volume** $dV = r^2\sin\theta drd\theta d\varphi$

Calculus of scalar and Vector

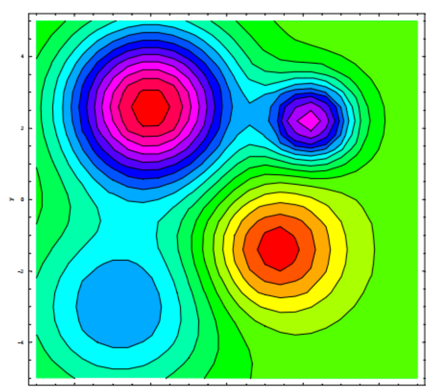
Gradient

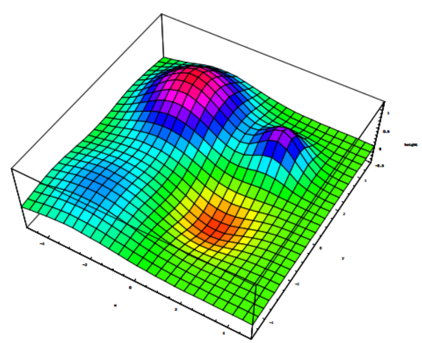
- Gradien dari suatu scalar field adalah suatu vektor yang **magnitudenya** menunjukkan perubahan maksimum scalar field tersebut dan **arahnya** menunjukkan arah dari peningkatan tercepat scalar field tersebut
- Ilustrasi 1



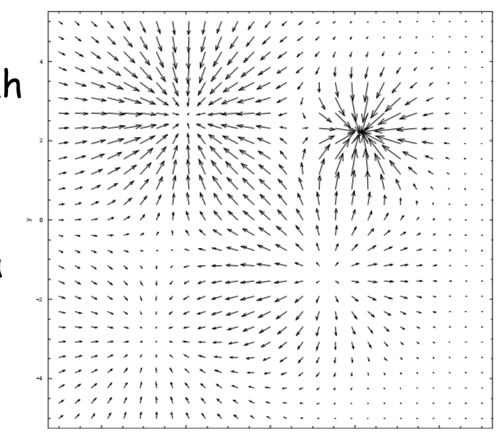
Suhu adalah scalar field. Jika terukur suhu pada suatu titik X dari sebuah sumber lilin, maka gradien terhadap suhu di X adalah vektor **A** dan bukan vektor **B**

- Ilustrasi 2





Ketinggian/kontur permukaan bumi adalah scalar field h(x,y). Jika kita melakukan operasi gradient terhadap h(x,y), maka Akan menghasilkan vector field seperti gambar disamping



Calculus of scalar and Vector

Gradient operator pada sistem koordinat

GRADIEN PADA KOORDINAT KARTESIAN

$$\nabla g(\bar{r}) = \frac{\partial g(\bar{r})}{\partial x}\hat{a}_x + \frac{\partial g(\bar{r})}{\partial y}\hat{a}_y + \frac{\partial g(\bar{r})}{\partial z}\hat{a}_z$$

GRADIEN PADA KOORDINAT TABUNG

$$\nabla g(\bar{r}) = \frac{\partial g(\bar{r})}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial g(\bar{r})}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial g(\bar{r})}{\partial \phi}\hat{a}_\phi$$

GRADIEN PADA KOORDINAT BOLA

$$\nabla g(\bar{r}) = \frac{\partial g(\bar{r})}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial g(\bar{r})}{\partial \phi}\hat{a}_\phi + \frac{\partial g(\bar{r})}{\partial z}\hat{a}_z$$

Contoh

misalkan:

$$\Leftrightarrow A(x,y,z) = x^2 \Leftrightarrow B(x,y,z) = -x^2 - y^2$$

$$\Leftrightarrow C(x,y,z) = x^2 y$$

maka:

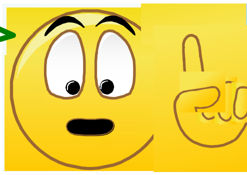
$$\vec{E} = \nabla A = \left[\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right]\left[x^2\right]$$

$$= \left[\frac{\partial x^2}{\partial x}\hat{a}_x + \frac{\partial x^2}{\partial y}\hat{a}_y + \frac{\partial x^2}{\partial z}\hat{a}_z\right]$$

$$= 2x\hat{a}_x$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

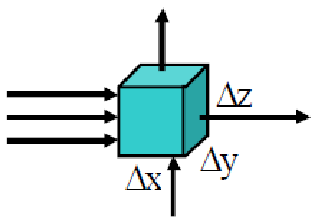
operator Del



Calculus of scalar and Vector

Divergensi

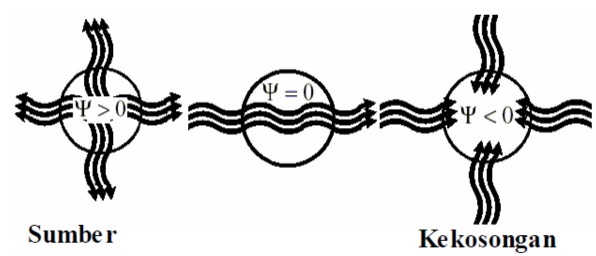
- Untuk mengestimasi dan meng-kuantisasi vector-vector field, sering dengan cara mengukur aliran vector field tersebut (atau netto aliran masuk dan keluar)
- Divergensi mengamati unsur volume tertentu yang sangat kecil, mengamati apakah ada '**sumber**' atau tidak di dalam volume tersebut
- Definisi dan simbol: Misalkan vector field yang diamati adalah vektor **D**



$$\lim_{\Delta V \to 0} \frac{\oint_S \vec{D} \bullet d\vec{S}}{\Delta V} \xrightarrow{\text{simbol}} \vec{\nabla} \bullet \vec{D}$$

- Hasil operasi divergensi adalah skalar, karena **dot product**
- Misalkan vector field yang diamati adalah vektor **D** maka:

  - **Hasil divergensi (+)** Jumlah vektor keluar > jumlah vektor masuk
    Artinya : Di dalam ruang **ada sumber**
  - **Hasil divergensi (-)** Jumlah vektor keluar < jumlah vektor masuk
    Artinya : **Ada kekosongan** dalam volume dan bersifat menyerap, contoh : **Black Hole**
  - **Hasil divergensi = 0** Jumlah vektor keluar = jumlah vektor masuk
    Artinya : **Tidak ada apa**-apa dalam volume tersebut



Calculus of scalar and Vector

Divergensi pada sistem koordinat

DIVERGENSI PADA KOORDINAT KARTESIAN

$$\nabla \cdot \mathbf{A}(\bar{r}) = \frac{\partial A_x(\bar{r})}{\partial x} + \frac{\partial A_y(\bar{r})}{\partial y} + \frac{\partial A_z(\bar{r})}{\partial z}$$

DIVERGENSI PADA KOORDINAT TABUNG

$$\nabla \cdot \mathbf{A}(\bar{r}) = \frac{1}{\rho}\left[\frac{\partial(\rho A_\rho(\bar{r}))}{\partial \rho}\right] + \frac{1}{\rho}\frac{\partial A_\phi(\bar{r})}{\partial \phi} + \frac{\partial A_z(\bar{r})}{\partial z}$$

DIVERGENSI PADA KOORDINAT BOLA

$$\nabla \cdot \mathbf{A}(\bar{r}) = \frac{1}{r^2}\left[\frac{\partial(r^2 A_r(\bar{r}))}{\partial r}\right] + \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta\, A_\theta(\bar{r}))}{\partial \theta}\right] + \frac{1}{r\sin\theta}\frac{\partial A_\phi(\bar{r})}{\partial \phi}$$

Contoh

$misalkan:$

$$\Leftrightarrow \vec{A}(x,y,z) = x\hat{a}_x \Leftrightarrow \vec{B}(x,y,z) = -x\hat{a}_x$$

$$\Leftrightarrow \vec{C}(x,y,z) = 5\hat{a}_y$$

$maka:$

$$E = \nabla \bullet \vec{A} = \left[\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right] \bullet [x\hat{a}_x]$$

$$= \left[\frac{\partial x}{\partial x}\hat{a}_x \bullet \hat{a}_x + \frac{\partial x}{\partial y}\hat{a}_y \bullet \hat{a}_x + \frac{\partial x}{\partial z}\hat{a}_z \bullet \hat{a}_x\right]$$

$$= 1$$

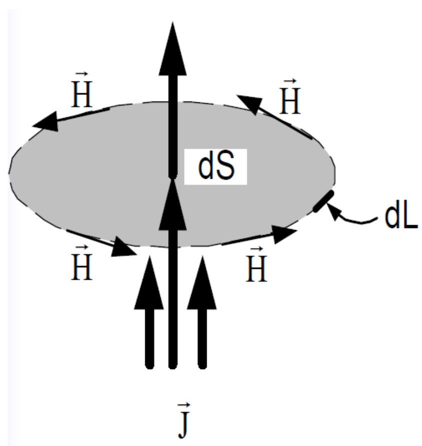
Calculus of scalar and Vector

Curl (Pusaran)

- Curl adalah ukuran seberapa besar putaran/pusaran/rotasi dari suatu vector field di sekitar titik tertentu
- Rotasi/pusaran terjadi jika adanya ketidakseragaman vector field
- Definisi dan simbol : Misalkan vector field yang diamati adalah vektor **H**

$$\lim_{\Delta S \to 0} \frac{\oint_L \vec{H} \bullet d\vec{L}}{\Delta S} \xrightarrow{\text{simbol}} \vec{\nabla} \times \vec{H}$$

- Curl adalah integral garis yang membatasi luas yang sangat kecil
- Curl digunakan untuk mengetahui vector field menembus permukaan diferensial yang sangat kecil, yang menyebabkan pusaran medan lain
- Ilustrasi



Rapat arus **J** yang menembus permukaan dS menimbulkan suatu pusaran/rotasi medan magnetik **H**

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

Calculus of scalar and Vector

Curl pada sistem koordinat

CURL PADA KOORDINAT KARTESIAN

$$\nabla \times \mathbf{A}(\bar{r}) = \left[\frac{\partial A_y(\bar{r})}{\partial z} - \frac{\partial A_z(\bar{r})}{\partial y}\right]\hat{a}_x + \left[\frac{\partial A_z(\bar{r})}{\partial x} - \frac{\partial A_x(\bar{r})}{\partial z}\right]\hat{a}_y + \left[\frac{\partial A_x(\bar{r})}{\partial y} - \frac{\partial A_y(\bar{r})}{\partial x}\right]\hat{a}_z$$

CURL PADA KOORDINAT BOLA

$$\nabla \times \mathbf{A}(\bar{r}) = \left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta A_\phi(\bar{r})\right) - \frac{1}{r\sin\theta}\frac{\partial A_\theta(\bar{r})}{\partial\phi}\right]\hat{a}_r + \left[\frac{1}{r\sin\theta}\frac{\partial A_r(\bar{r})}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}\left(r A_\phi(\bar{r})\right)\right]\hat{a}_\theta + \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r A_\theta(\bar{r})\right) - \frac{1}{r}\frac{\partial A_r(\bar{r})}{\partial\theta}\right]\hat{a}_\phi$$

CURL PADA KOORDINAT TABUNG

$$\nabla \times \mathbf{A}(\bar{r}) = \left[\frac{1}{\rho}\frac{\partial A_z(\bar{r})}{\partial\phi} - \frac{\partial A_\phi(\bar{r})}{\partial z}\right]\hat{a}_\rho + \left[\frac{\partial A_\rho(\bar{r})}{\partial z} - \frac{\partial A_z(\bar{r})}{\partial\rho}\right]\hat{a}_\phi + \left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho A_\phi(\bar{r})\right) - \frac{1}{\rho}\frac{\partial A_\rho(\bar{r})}{\partial\phi}\right]\hat{a}_z$$

Calculus of scalar and Vector

Curl pada sistem koordinat

Contoh

$$misalkan:$$

$$\Leftrightarrow \vec{A}(x,y,z) = -y\hat{a}_x + x\hat{a}_y \Leftrightarrow \vec{B}(x,y,z) = y\hat{a}_x + x\hat{a}_y$$

$$maka:$$

$$E = \nabla \times \vec{A} = \left[\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right] \times \left[-y\hat{a}_x + x\hat{a}_y\right]$$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix} \begin{matrix} \hat{a}_x & \hat{a}_y \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ -y & x \end{matrix}$$

$$= \left(\frac{\partial 0}{\partial y}\hat{a}_x - \frac{\partial x}{\partial z}\hat{a}_x\right) + \left(\frac{-\partial y}{\partial z}\hat{a}_y - \frac{\partial 0}{\partial x}\hat{a}_y\right) + \left(\frac{\partial x}{\partial x}\hat{a}_z - \frac{-\partial y}{\partial y}\hat{a}_z\right)$$

$$= 0 + 0 + \left(\frac{\partial x}{\partial x}\hat{a}_z - \frac{-\partial y}{\partial y}\hat{a}_z\right) = 1 - (-1)\hat{a}_z = 2\hat{a}_z$$

Calculus of scalar and Vector

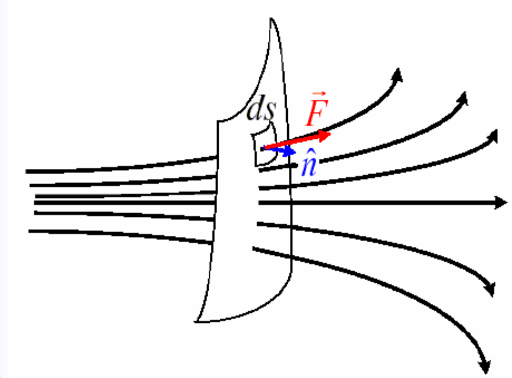
Berbagai Identitas Vektor

$\vec{\nabla}\left(\vec{A}.\vec{B}\right) \equiv \left(\vec{A}.\vec{\nabla}\right)\vec{B} + \left(\vec{B}.\vec{\nabla}\right)\vec{A} + \vec{A}\times\left(\vec{\nabla}\times\vec{B}\right) + \vec{B}\times\left(\vec{\nabla}\times\vec{A}\right)$	$\vec{\nabla}\times\vec{\nabla}\times\vec{A} \equiv \vec{\nabla}\left(\vec{\nabla}.\vec{A}\right) - \nabla^2\vec{A}$
$\vec{\nabla}\times\left(\vec{A}\times\vec{B}\right) \equiv \vec{A}\vec{\nabla}.\vec{B} - \vec{B}\vec{\nabla}.\vec{A} + \left(\vec{B}.\vec{\nabla}\right)\vec{A} - \left(\vec{A}.\vec{\nabla}\right)\vec{B}$	$\vec{\nabla}(V + W) \equiv \vec{\nabla}V + \vec{\nabla}W$
$\vec{A}\times\left(\vec{B}\times\vec{C}\right) \equiv \left(\vec{A}.\vec{C}\right)\vec{B} - \left(\vec{A}.\vec{B}\right)\vec{C}$	$\vec{\nabla}.\vec{\nabla}\times\vec{A} \equiv 0$
$\left(\vec{A}\times\vec{B}\right)\vec{C} \equiv \left(\vec{B}\times\vec{C}\right)\vec{A} \equiv \left(\vec{C}\times\vec{A}\right)\vec{B}$	$\vec{\nabla}.\left(V\vec{A}\right) \equiv \vec{A}.\vec{\nabla}V + V\vec{\nabla}.\vec{A}$
$\nabla\times\left(V\vec{A}\right) \equiv \vec{\nabla}V\times\vec{A} + V\vec{\nabla}\times\vec{A}$	$\vec{\nabla}(VW) \equiv V\vec{\nabla}W + W\vec{\nabla}V$
$\vec{\nabla}.\left(\vec{A}\times\vec{B}\right) \equiv \vec{B}.\vec{\nabla}\times\vec{A} - \vec{A}.\vec{\nabla}\times\vec{B}$	$\vec{\nabla}.\vec{\nabla}V \equiv \nabla^2 V$
$\vec{\nabla}.\left(\vec{A}+\vec{B}\right) \equiv \vec{\nabla}.\vec{A} + \vec{\nabla}.\vec{B}$	$\vec{\nabla}\times\vec{\nabla}V \equiv 0$
$\vec{\nabla}\times\left(\vec{A}+\vec{B}\right) \equiv \vec{\nabla}\times\vec{A} + \vec{\nabla}\times\vec{B}$	

Divergence Theorem/Gauss's Theorem

- Teorema Divergensi

$$\iiint_v \left(\nabla \bullet \vec{F}\right) dV = \oiint_s \left(\vec{F} \bullet d\vec{S}\right)$$

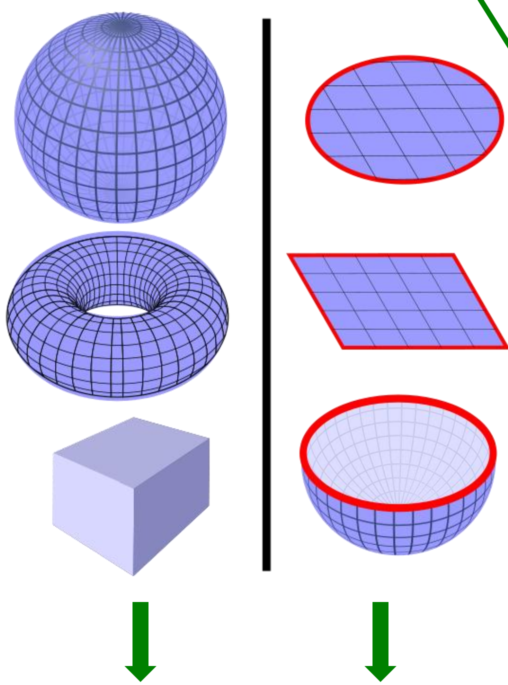


FLUX: adalah netto aliran yang menembus permukaan dengan arah normal terhadap permukaan

$$\Psi = \iint_S \vec{F} \bullet d\vec{S} = \iint_S F\cos\theta \, dS$$

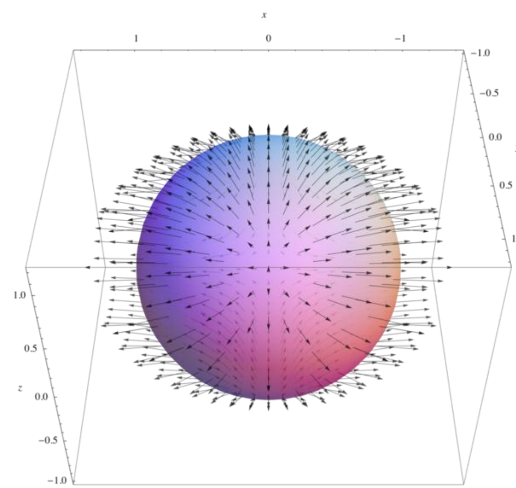
$$d\vec{S} = dS\,\hat{n}$$

Vektor dS selalu tegaklurus terhadap elemen permukaan dS



Close Surface **Open Surface**

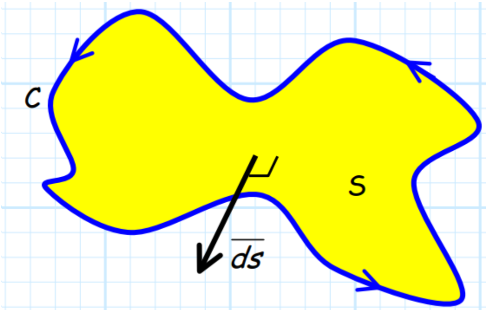
Teorema divergensi menyatakan bahwa jumlah flux dari suatu vector field yang menembus suatu permukaan tertutup/close surface sama dengan jumlah total / integral volume dari suatu divergensi vector field tersebut pada seluruh area(volume) yang terlingkupi close surface tersebut



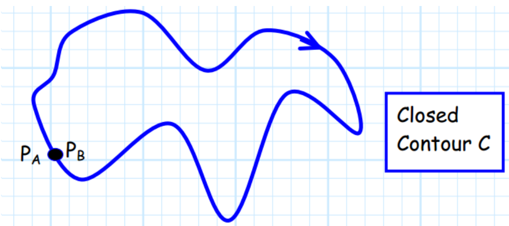
Stoke's Theorem

- Theorema stoke

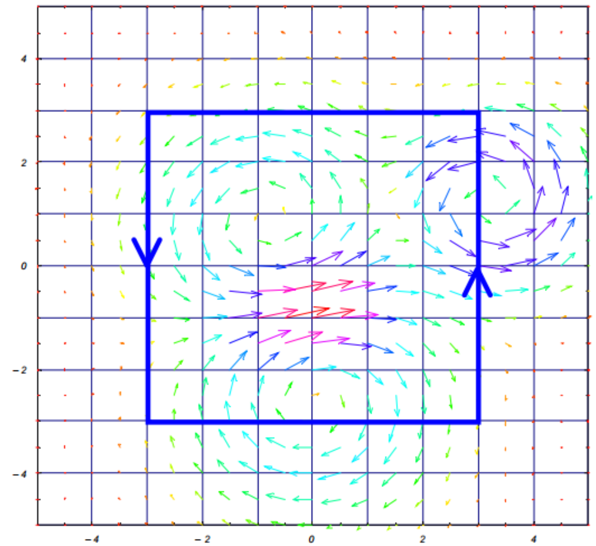
$$\iint_S \nabla \times \mathbf{A}(\bar{r}) \cdot \overline{ds} = \oint_C \mathbf{A}(\bar{r}) \cdot \overline{d\ell}$$



Contour/Garis tepi **C** adalah close contour yang mengelilingi permukaan/surface **S**. arah dari contour **C** didefinisikan dari vektor **ds** dan aturan putaran tangan kanan. Pada gambar disamping contour **C** berputar berlawanan jarum jam.



A closed contour adalah contour/garis tepi yang **dimulai dan diakhiri** pada **titik yang sama**!



Artinya, jika kita ingin mengintegralkan suatu vector field yang menembus suatu **open surface** atau seolah olah kita ingin menjumlahkan / mengintegralkan suatu pusaran di setiap titik pada suatu permukaan.

$$\iint_S \mathbf{B}(\bar{r}) \cdot \overline{ds} = \iint_S \nabla \times \mathbf{A}(\bar{r}) \cdot \overline{ds}$$

Hal tersebut sama saja dengan secara sederhana menjumlahkan /mengintegralkan 'rotasi' vector field tersebut pada sepanjang **close contour** yang mengelilingi surface tersebut.

ANY QUESTION???


Thank you