MODULASI PSK (PHASE SHIFT KEYING)

Sistem Komunikasi
Prodi D3 TT
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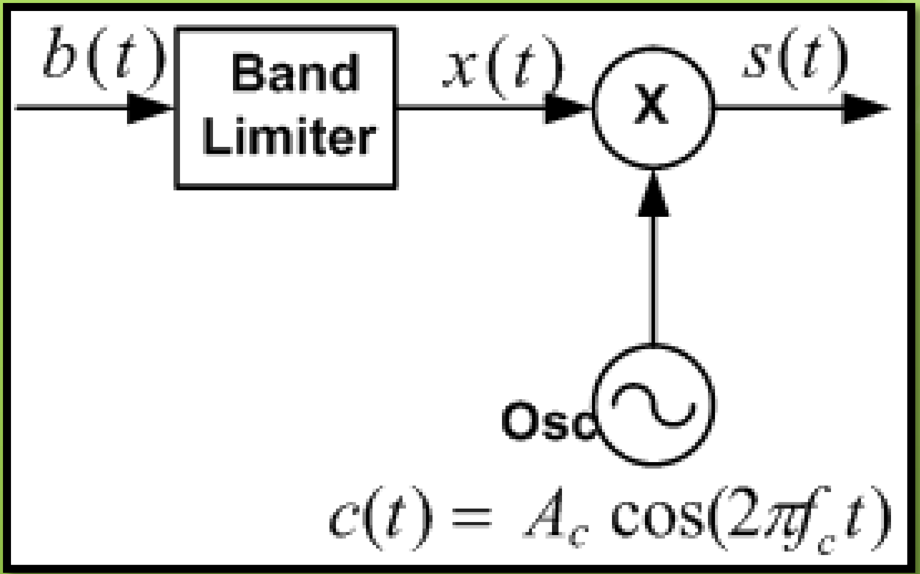
BINARY SHIFT KEYING

Binary Shift Keying (BPSK)

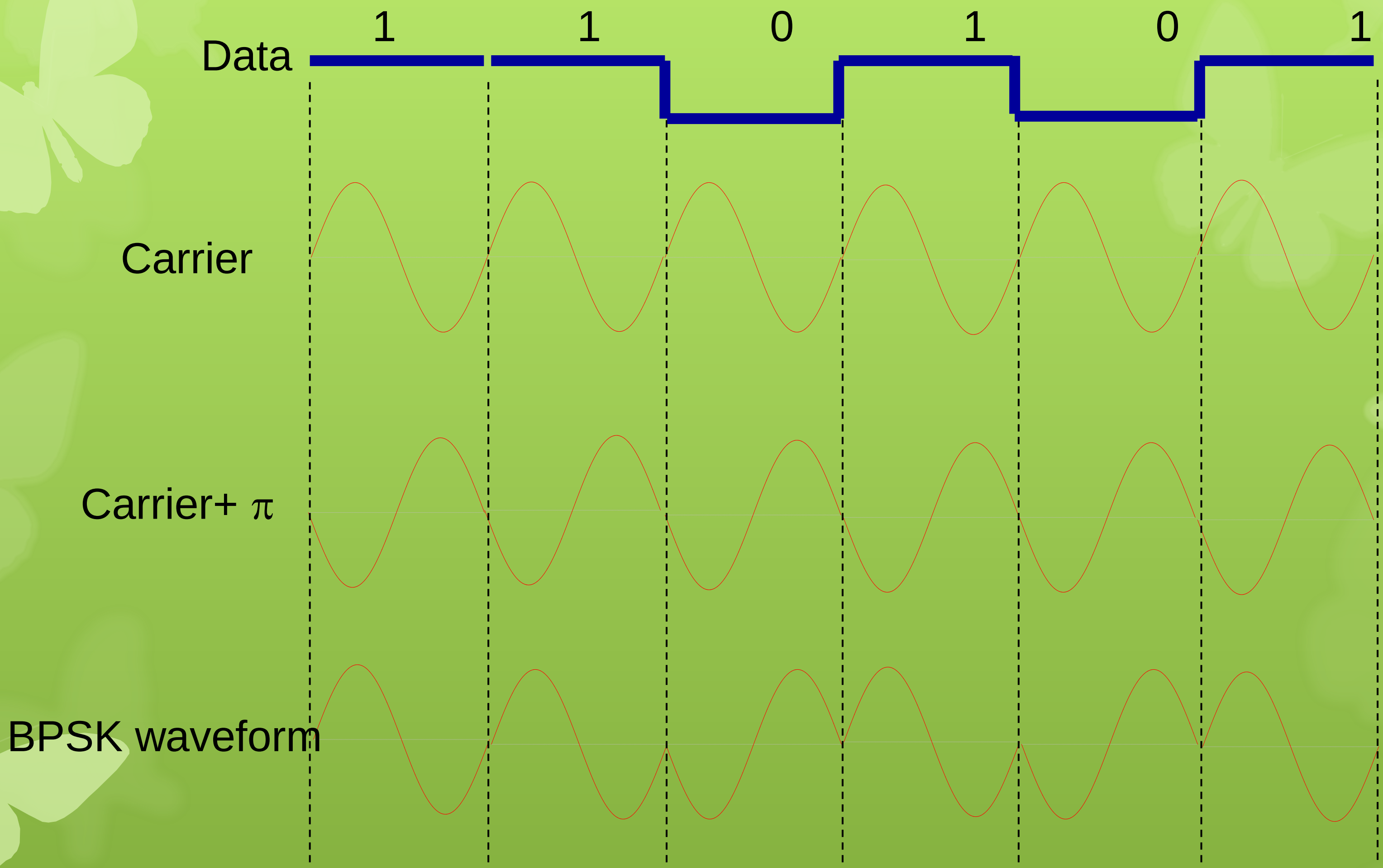
- Perubahan Parameter Fasa dari sinyal pembawa sesuai dengan sinyal informasi.
- Menggunakan alternatif-alternatif fasa gelombang sinus utk mengkodekan bit-bit dimana Fasa dipisahkan 180 derajat
- Sederhana utk diimplementasikan, tidak efisien dalam penggunaan *bandwidth*.
- Sangat kokoh, sering digunakan secara extensif pada komunikasi satelit.

Blok Sistem BPSK



$$s(t) = \begin{cases} A\cos(2\pi f_c t + 0)\,;\, b(t) = "1" \\ A\cos(2\pi f_c t + \pi)\,;\, b(t) = "0" \end{cases}$$

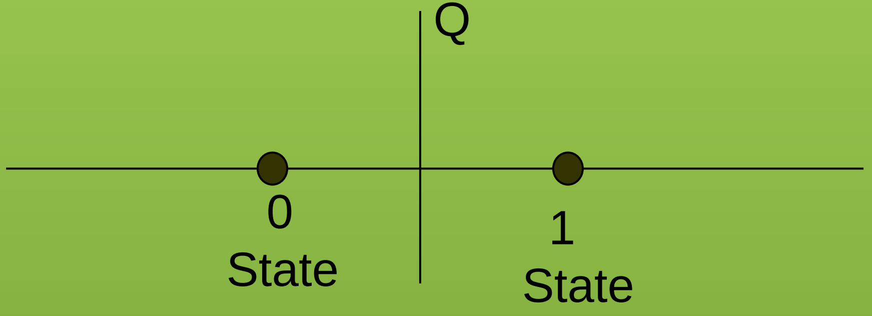
Contoh Sinyal BPSK



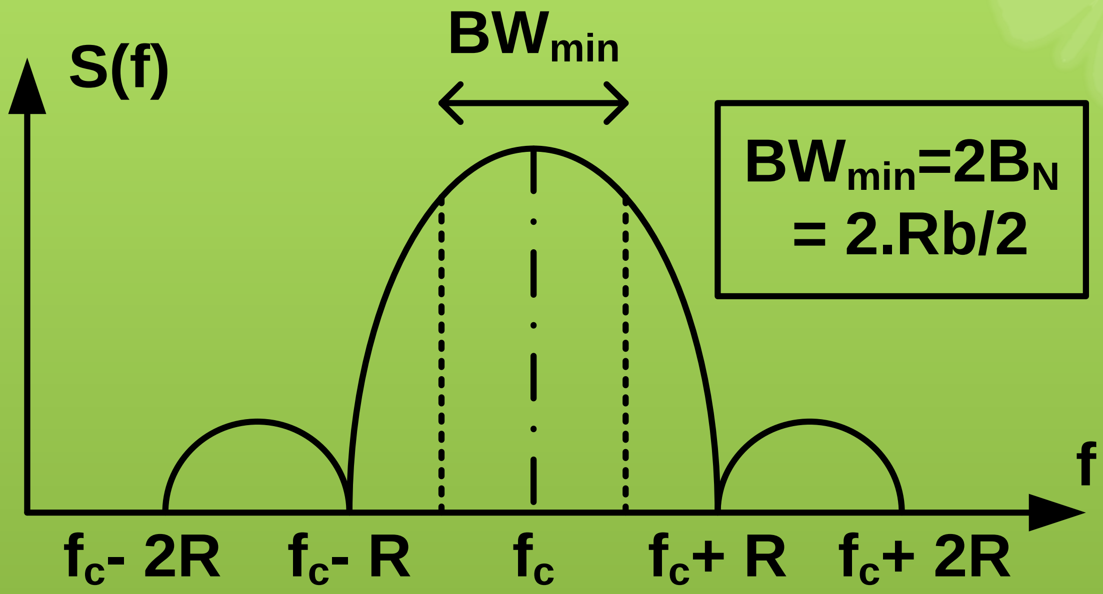
Konstelasi sinyal BPSK

- Fasa dipisahkan 180 derajat.

$$s_1(t) = A_c \cos(2\pi f_c t) \qquad \text{binary '1'}$$
$$s_2(t) = A_c \cos(2\pi f_c t + \pi) \quad \text{binary '0'}$$



Spektrum Sinyal BPSK

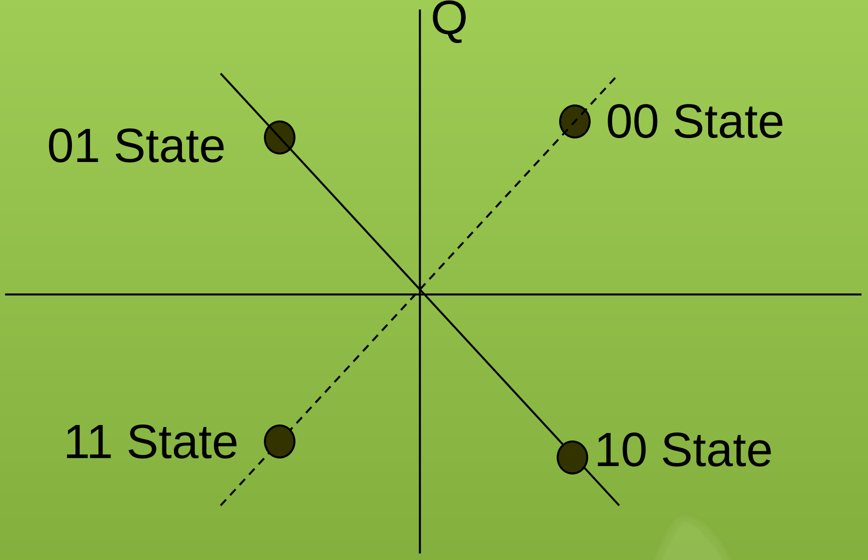


B_N=Bandwidth Nyquist

MODULASI QPSK (QUADRATURE SHIFT KEYING)

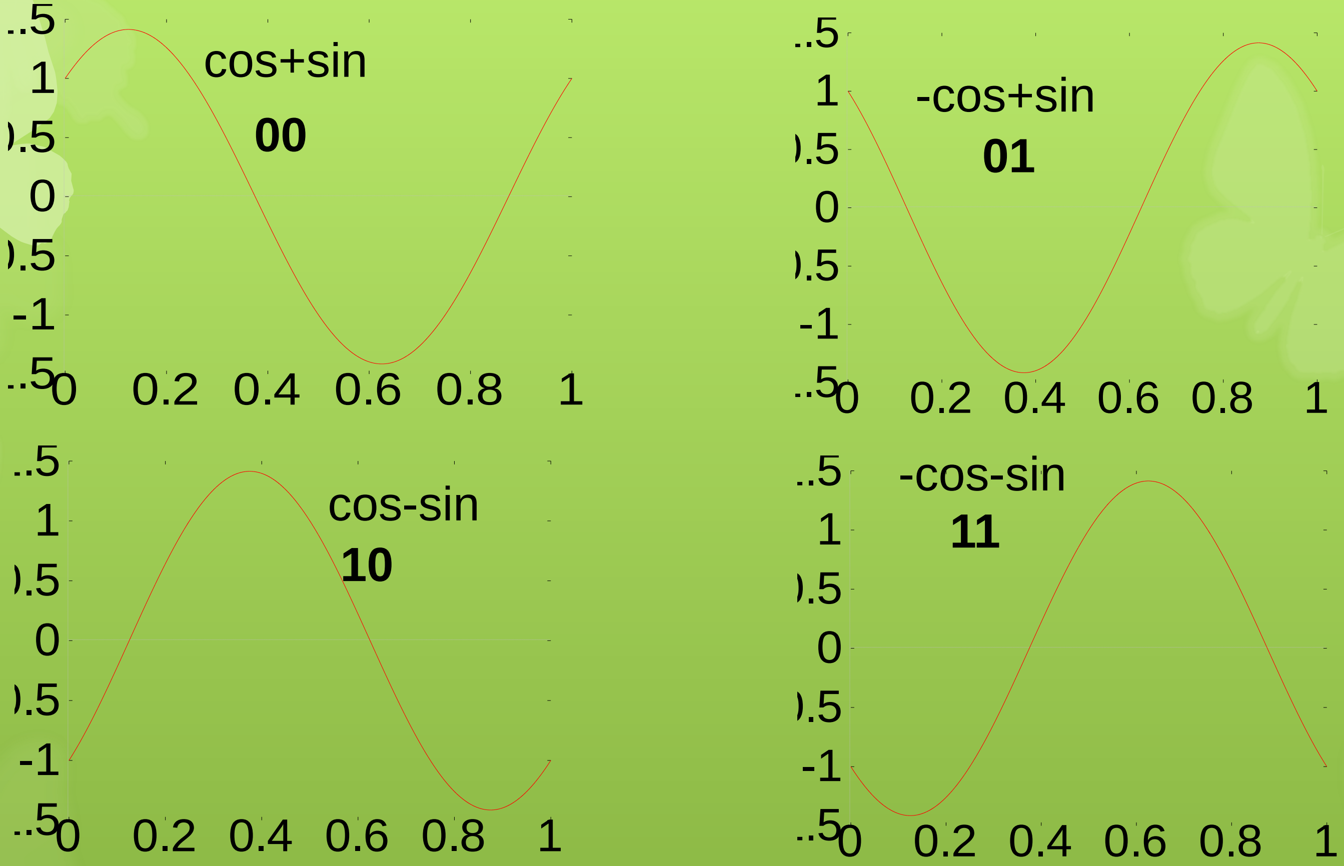
Quadrature Phase Shift Keying

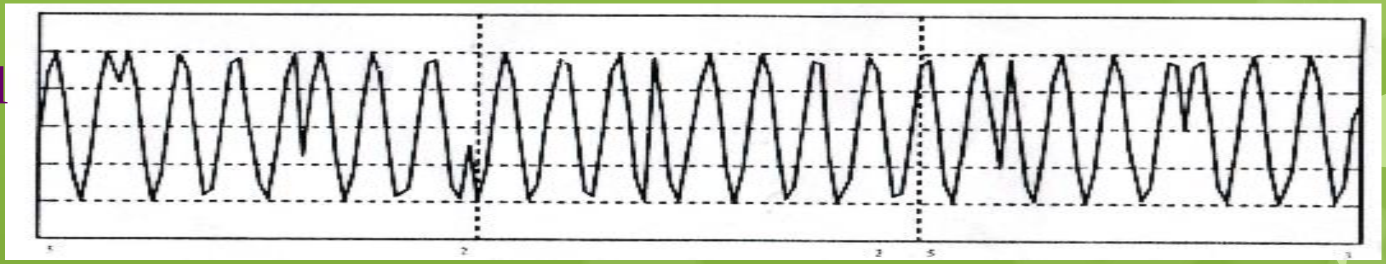
- Teknik modulasi multilevel : 2 bit per symbol
- Lebih efisien spektrum, lebih kompleks receiver.
- Dua kali lebih efisien bandwidth daripada BPSK



Phase of Carrier: $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$

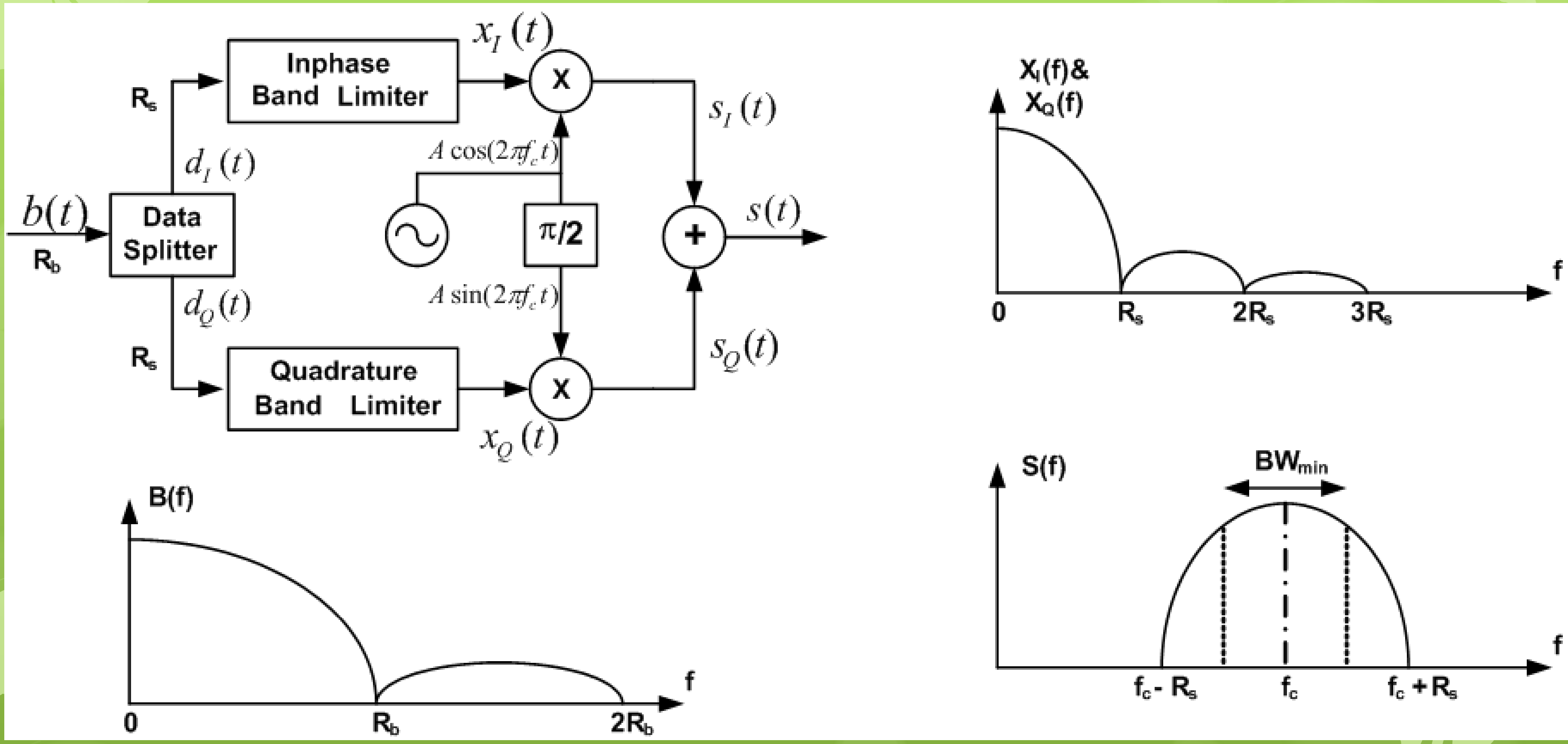
4 bentuk gelombang berbeda:





Bentuk Sinyal QPSK

Blok Sistem QPSK



Bandpass modulation: Signal Space & Vector

- Bandpass modulation: The process of converting data signal to a sinusoidal waveform where its amplitude, phase or frequency, or a combination of them, is varied in accordance with the transmitting data.
- Bandpass signal (**General Condition**):

$$s_i(t) = h(t)\sqrt{\frac{2E_i}{T}}\cos\left(\omega_c t + (i-1)\Delta\omega t + \phi_i(t)\right) \quad 0 \le t \le T$$

where $h(t)$ is the baseband pulse shape with energy E_h.

- We assume here (otherwise will be stated):
  - $h(t)$ is a rectangular pulse shape with unit energy.
  - Gray coding is used for mapping bits to symbols.
  - E_s denotes average symbol energy given by $E_s = \frac{1}{M}\sum_{i=1}^{M} E_i$

Phase Shift Keying (PSK)

I. PSK signal waveform (transmitted signal):

$$s_i(t)=\sqrt{\frac{2E}{T}}\cos[\omega_0 t+\phi_i(t)],\quad \text{phase: } \phi_i(t)=\frac{2\pi i}{M},\quad 0\le t\le T,\quad i=1,\cdots,M,$$

- Phase (examples):

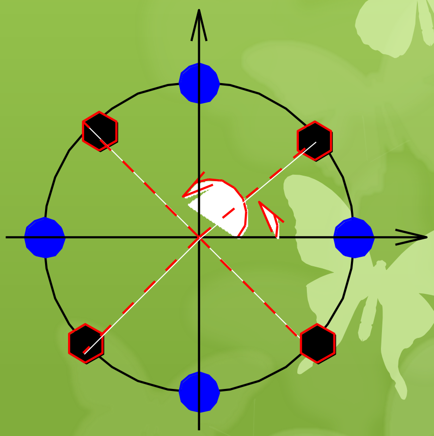
$$\text{BPSK}(M=2):\quad \phi_i\in\{\pi,0\}$$

$$\text{QPSK}(M=4):\quad \phi_i\in\left\{\frac{\pi}{2},\pi,\frac{3\pi}{2},0\right\}\ \text{or}\ \left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}_{i=1/2}^{7/2}$$

- Symbol energy and symbol interval

E is symbol energy. T is symbol interval.

Can you show that $E=\int_0^T s_i^2(t)dt$?



II. Signal space representation

Note: decomposition of PSK signal waveform:

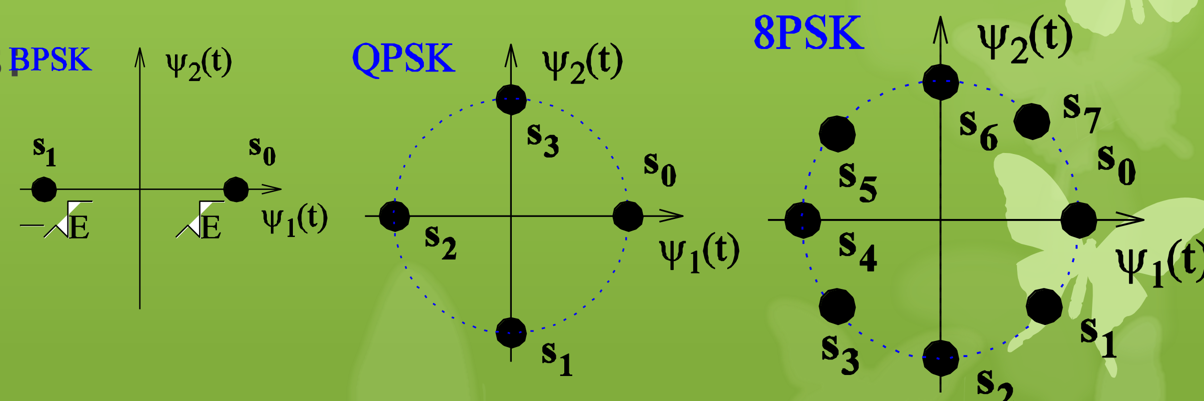
$$s_i(t) = \sqrt{\frac{2E}{T}}\cos[\phi_i(t)]\cos(\omega_0 t) - \sqrt{\frac{2E}{T}}\sin[\phi_i(t)]\sin(\omega_0 t)$$

1) Orthonormal basis: $\psi_1(t) = \sqrt{\frac{2}{T}}\cos(\omega_0 t), \qquad \psi_2(t) = \sqrt{\frac{2}{T}}\sin(\omega_0 t)$

2) Signal vector: $s_i(t): \ \mathbf{s}_i = \left(\sqrt{E}\cos\frac{2\pi i}{M}, \quad -\sqrt{E}\sin\frac{2\pi i}{M}\right), \quad i = 1, \cdots, M$

3) Constellations:

(examples)



Signal space representation (cont.)

4) A proof of signal space representation

- Bases are orthonormal

$$\int_{-\infty}^{\infty}\psi_1^2(t)dt=\int_0^T 2/T\cos^2(\omega_0 t)dt=2/T\int_0^T[1+\cos(2\omega_0 t)]/2dt=1.$$

$$\int_{-\infty}^{\infty}\psi_2^2(t)dt=\int_0^T 2/T\sin^2(\omega_0 t)dt=2/T\int_0^T[1-\cos(2\omega_0 t)]/2dt=1.$$

$$\int_{-\infty}^{\infty}\psi_1(t)\psi_2(t)dt=\int_0^T 2/T\cos(\omega_0 t)\sin(\omega_0 t)dt=2/T\int_0^T\sin(2\omega_0 t)/2dt=0.$$

- Signal space vector for each waveform $s_i(t)$

$$a_{i1}=\int_{-\infty}^{\infty}s_i(t)\psi_1(t)dt=2\sqrt{E}/\mathrm{T}\int_0^T\cos[\omega_0 t+\phi_i(t)]\cos(\omega_0 t)dt$$
$$=\sqrt{E}/\mathrm{T}\int_0^T\cos[\phi_i(t)]+\cos[2\omega_0 t+\phi_i(t)]dt=\sqrt{E}\cos[\phi_i(t)]$$

$$a_{i2}=\int_{-\infty}^{\infty}s_i(t)\psi_2(t)dt=2\sqrt{E}/\mathrm{T}\int_0^T\cos[\omega_0 t+\phi_i(t)]\sin(\omega_0 t)dt$$
$$=\sqrt{E}/\mathrm{T}\int_0^T-\sin[\phi_i(t)]+\sin[2\omega_0 t+\phi_i(t)]dt=-\sqrt{E}\sin[\phi_i(t)]$$

Signal space representation (cont.)

5) What happens if baseband pulse-shaping ***h(t)*** is considered?

- Signal waveform:

$$s_i(t) = \sqrt{\frac{2E}{T}} h(t) \cos[\omega_0 t + \phi_i(t)]$$

- Use basis (note that h(t) can be assumed normalized):

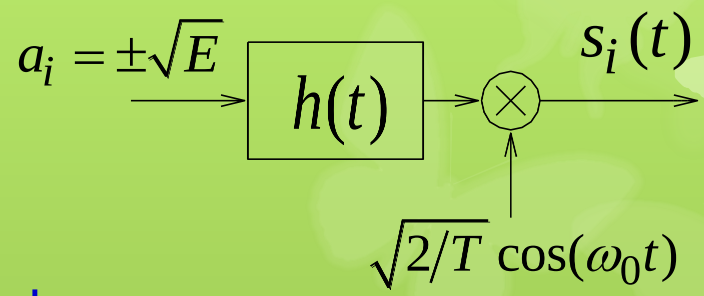
$$\psi_1(t) = \sqrt{\frac{2}{T}} h(t) \cos(\omega_0 t), \qquad \psi_2(t) = \sqrt{\frac{2}{T}} h(t) \sin(\omega_0 t)$$

- Signal space vector is still:

$$s_i(t): \quad \mathbf{s}_i = \left(\sqrt{E} \cos \frac{2\pi i}{M}, \quad -\sqrt{E} \sin \frac{2\pi i}{M} \right), \quad i = 1, \cdots, M$$

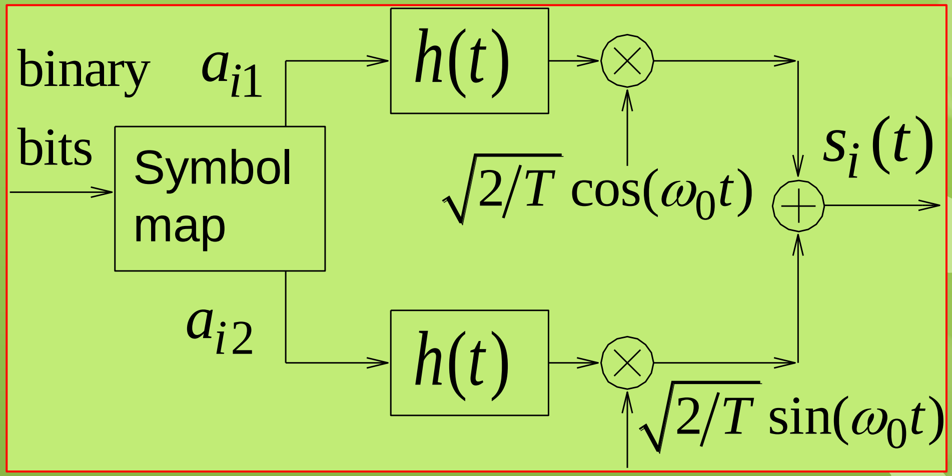
- Conclusion: there is no difference in signal space whether pulse-shaping is considered. We can study only PSK instead of the more general PM.

PSK modulator



- Special case: BPSK modulator

- e: M



Note:

Inputs are signal-space vector.

Carriers are in basis form.

$$s_i(t) = a_{i1}\sqrt{2/T}\cos(\omega_0 t) + a_{i2}\sqrt{2/T}\sin(\omega_0 t)$$

$$\mathbf{s}_i = (a_{i1}, a_{i2}) = \left(\sqrt{E}\cos(2\pi i/M),\ -\sqrt{E}\sin(2\pi i/M)\right)$$

- Just like DSB modulation:
  ## Bandwidth of PSK signal waveform

$$W_{\text{PSK}} = 2W_{\text{baseband}}$$

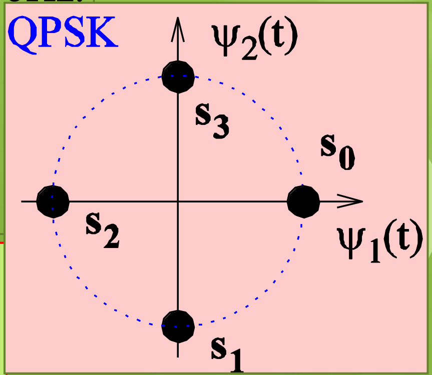
- **Exercise :** Consider QPSK transmission with date rate 2000 bps. The magnitude of the signal si(t) is √2E/T =1 volt.
  a) What is the minimum PSK signal bandwidth?
  b) Find the signal space points
  c) Draw the constellation
  d) Find signal waveform for transmitting {1001}.

a) $R_s = R_b/(\log_2 M) = 2000/2 = 1000.\ \ W_{\text{PSK}} = 2W_{\text{baseband,min}} = 2R_s/2 = 1000\text{Hz}.$

b) $\mathbf{s}_i = (\sqrt{E}\cos 2\pi i/4, -\sqrt{E}\sin 2\pi i/4)$, where $E = T/2 = 0.5\times 10^{-3}, \quad i = 1, \cdots,$

d) Define mapping as: $\{00{:}0,\ 01{:}\pi,\ 10{:}\pi/2,\ 11{:}3\pi/2\}$.

Then $\{10\} \rightarrow s_1(t) = \cos(\omega_0 t + \pi/2). \quad \{01\} \rightarrow s_2(t) = \cos(\omega_0 t + \pi)$

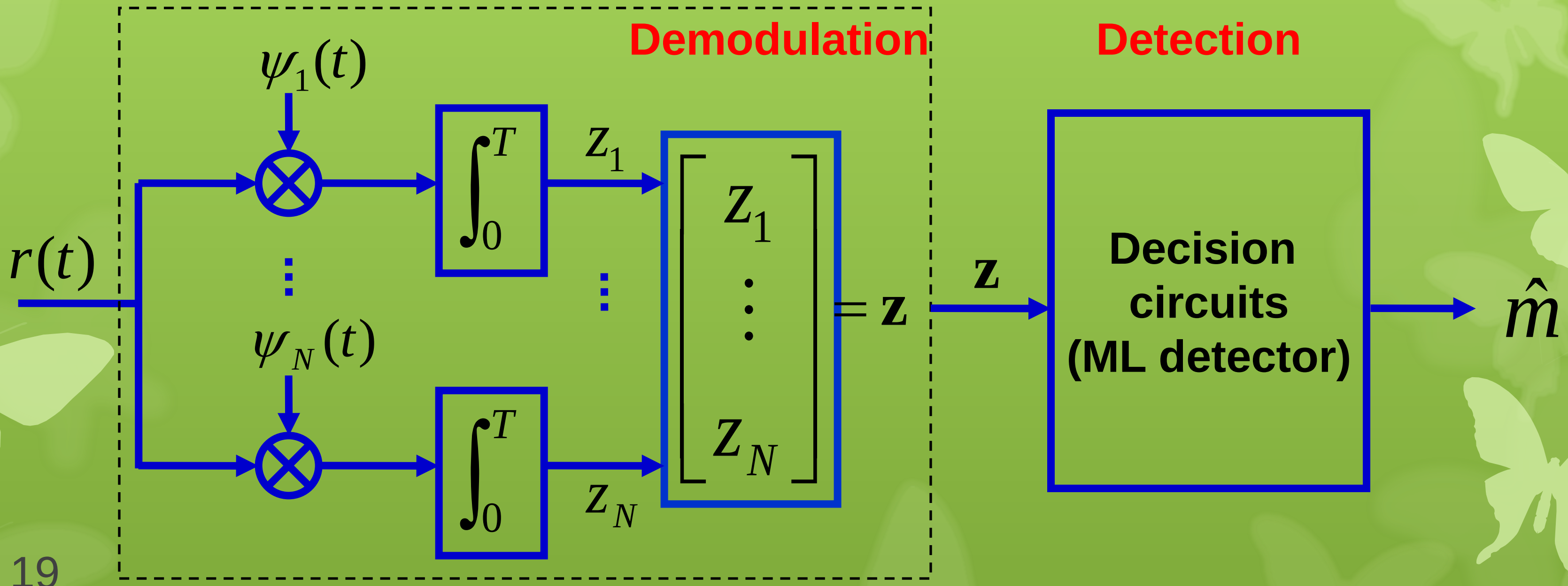


Phase $\phi_i(t)$ in $s_i(t)$ is different from phase of $\mathbf{s}_i$ (phase in signal space)

Demodulation and detection

Demodulation: The receiver signal is converted to baseband, filtered and sampled.

Detection: Sampled values are used for detection using a decision rule such as ML detection rule.



Demodulations type:

- Some notations
  - Carrier: $s(t) = A(t)\cos[\omega_0 t + \phi(t)], \qquad \omega_0 = 2\pi f_0$
  - Modulation types with respect to carrier parameters

Modulation	Varying parameter	Demodulation
PSK	$\phi(t)$	Coherent or noncoherent
QAM	both $A(t)$ and $\phi(t)$	Coherent or noncoherent
FSK	ω_0	Coherent or Noncoherent

Two dimensional modulation, demodulation and detection (M-PSK)

$$s_i(t) = \sqrt{\frac{2E_s}{T}}\cos\left(\omega_c t + \frac{2\pi i}{M}\right)$$

- M-ary Phase Shift Keying (M-PSK)

$$s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) \quad i = 1, \dots, M$$

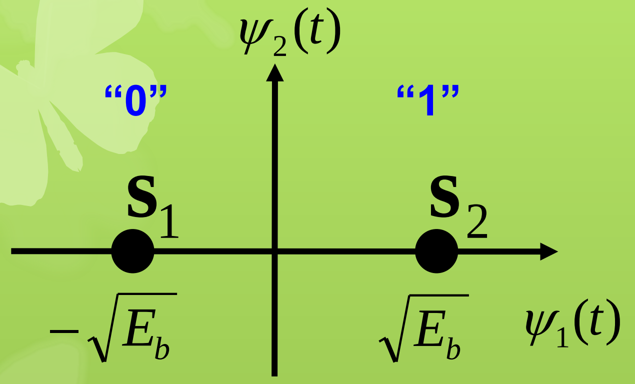
$$\psi_1(t) = \sqrt{\frac{2}{T}}\cos(\omega_c t) \quad \psi_2(t) = \sqrt{\frac{2}{T}}\sin(\omega_c t)$$

$$a_{i1} = \sqrt{E_s}\cos\left(\frac{2\pi i}{M}\right) \qquad a_{i2} = \sqrt{E_s}\sin\left(\frac{2\pi i}{M}\right)$$

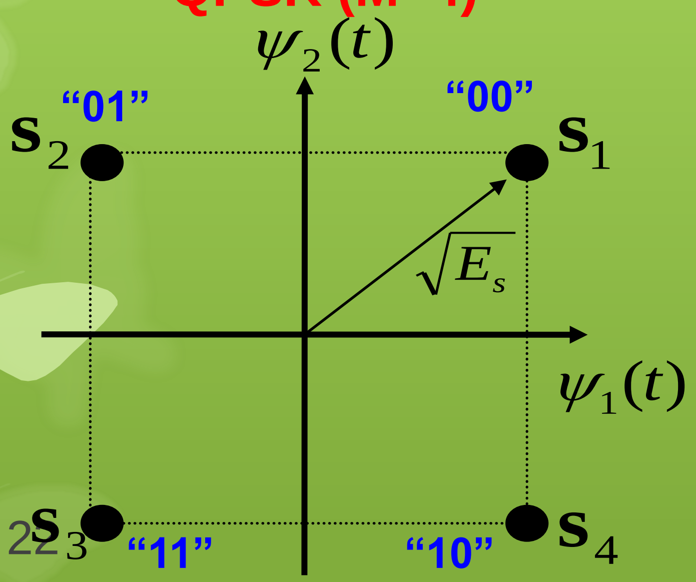
$$E_s = E_i = \|\mathbf{s}_i\|^2$$

Two dimensional mod… (MPSK)

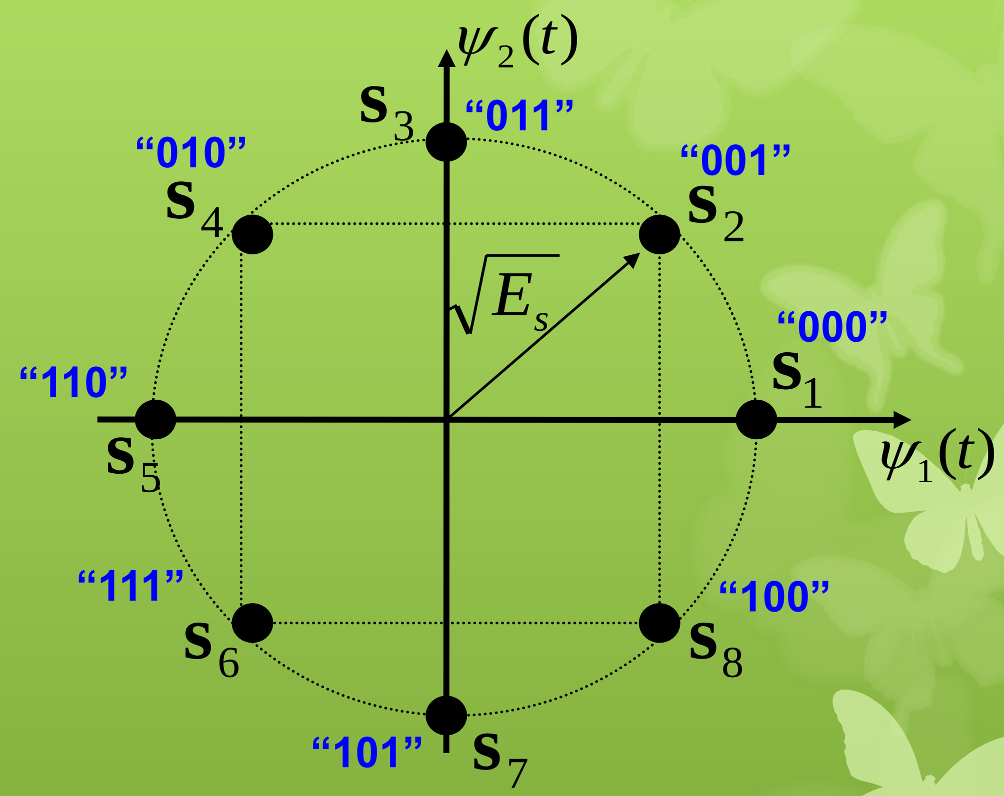
BPSK (M=2)



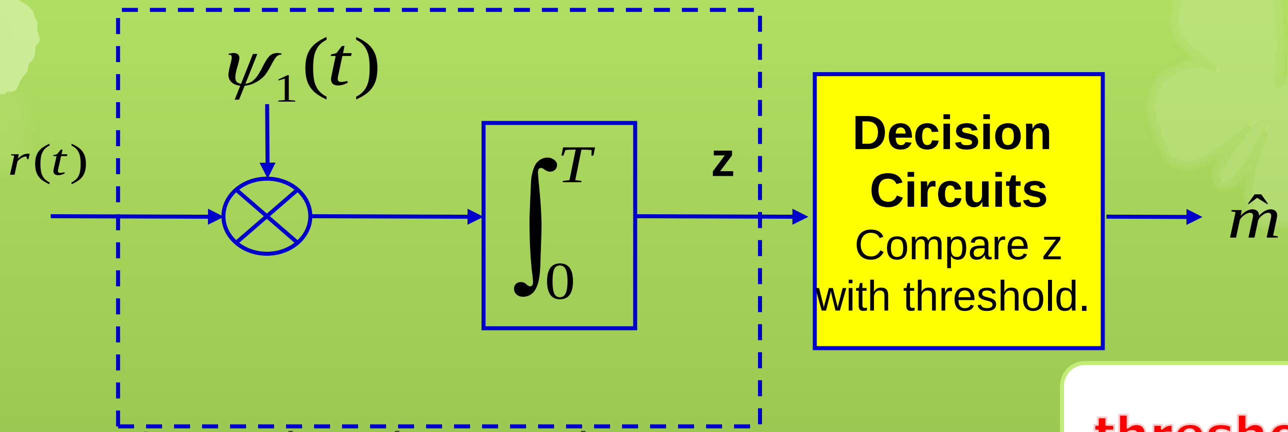
QPSK (M=4)



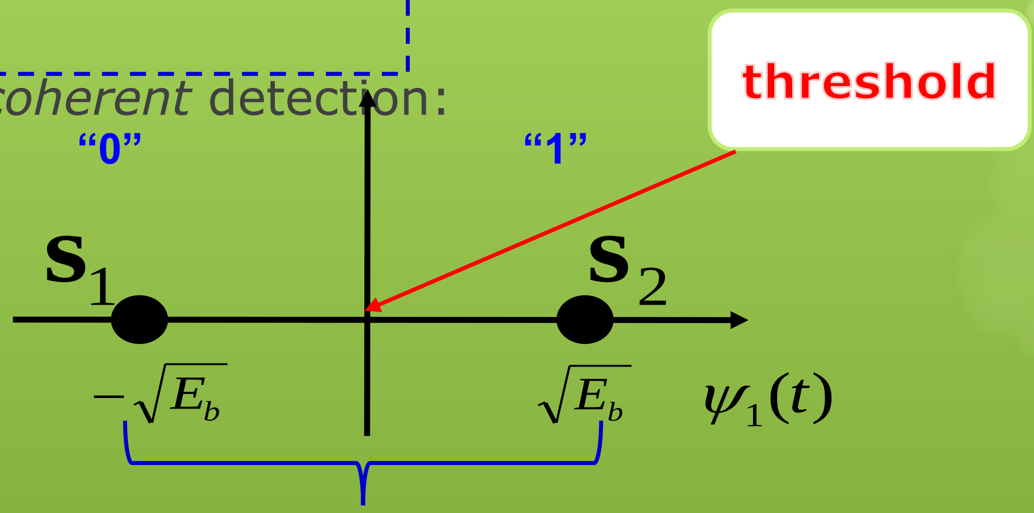
8PSK (M=8)



Demodulation BPSK



- BPSK with *coherent* detection:



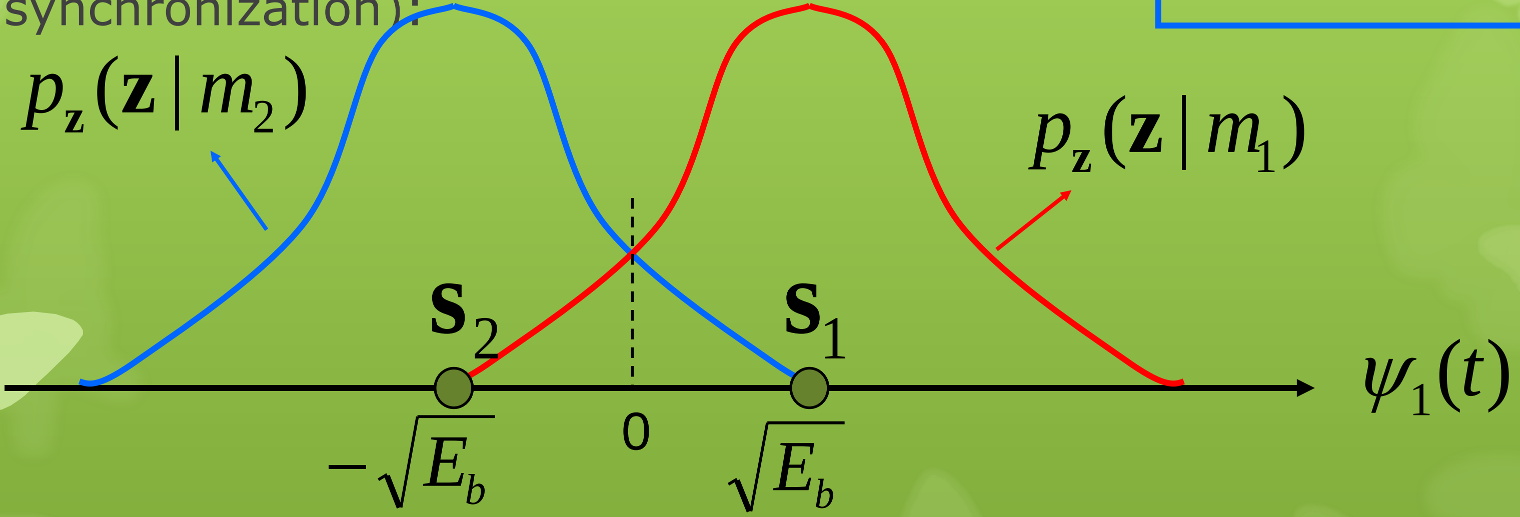
$$\|\mathbf{s}_1 - \mathbf{s}_2\| = 2\sqrt{E_b}$$

Error probability …

$$P_B = Q\left(\frac{\|\mathbf{s}_1 - \mathbf{s}_2\| / 2}{\sqrt{N_0 / 2}}\right)$$

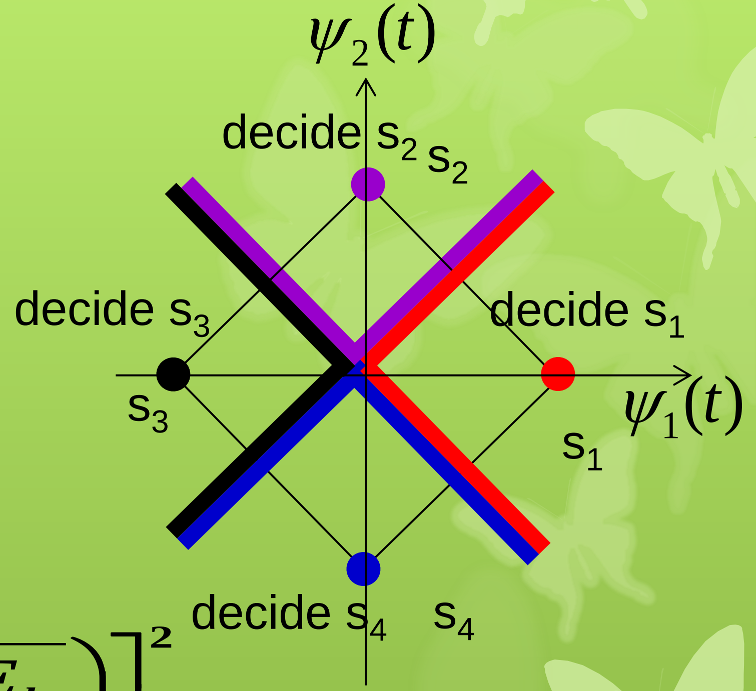
$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- BPSK with *coherent* detection (with perfect carrier synchronization):



Demodulation M-PSK

Decision Region QPSK



- Coherent detection of Q-PSK

$$p_C = \left(1 - p_{BPSK-I}\right)^2 = \left[1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]^2$$

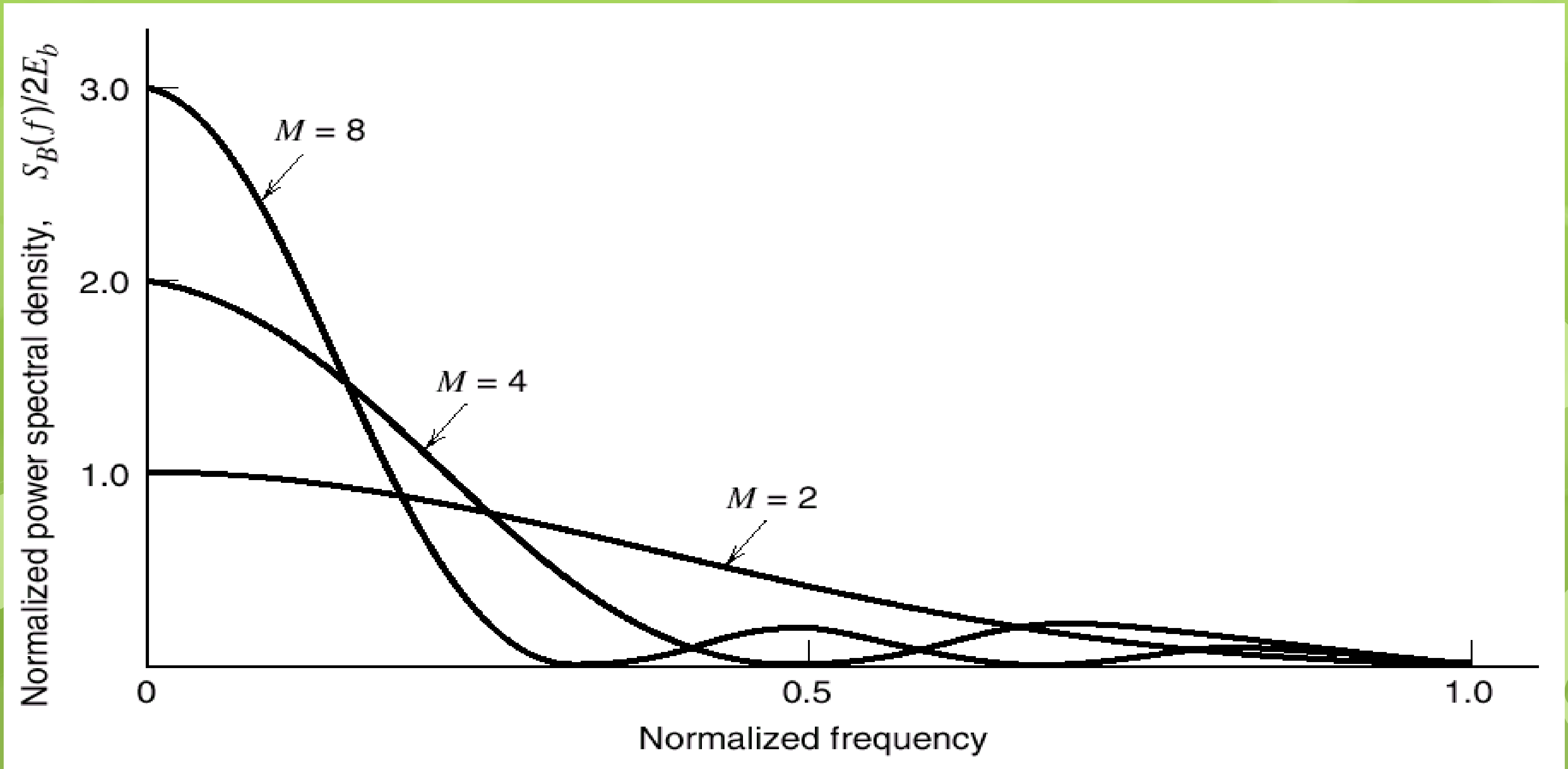
$$p_e = 1 - p_c = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1 - \frac{1}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]$$

$$p_e \approx Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Power Spectra of M-Ary PSK

$$S_B(f) = 2E \operatorname{sinc}^2(Tf)$$

$$S_B(f) = 2E_b \log M \operatorname{sinc}^2(T_b f \log_2 M)$$



QPSK vs. BPSK

- Let's compare the two based on BER and bandwidth

BER

BPSK	QPSK
$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

EQUAL

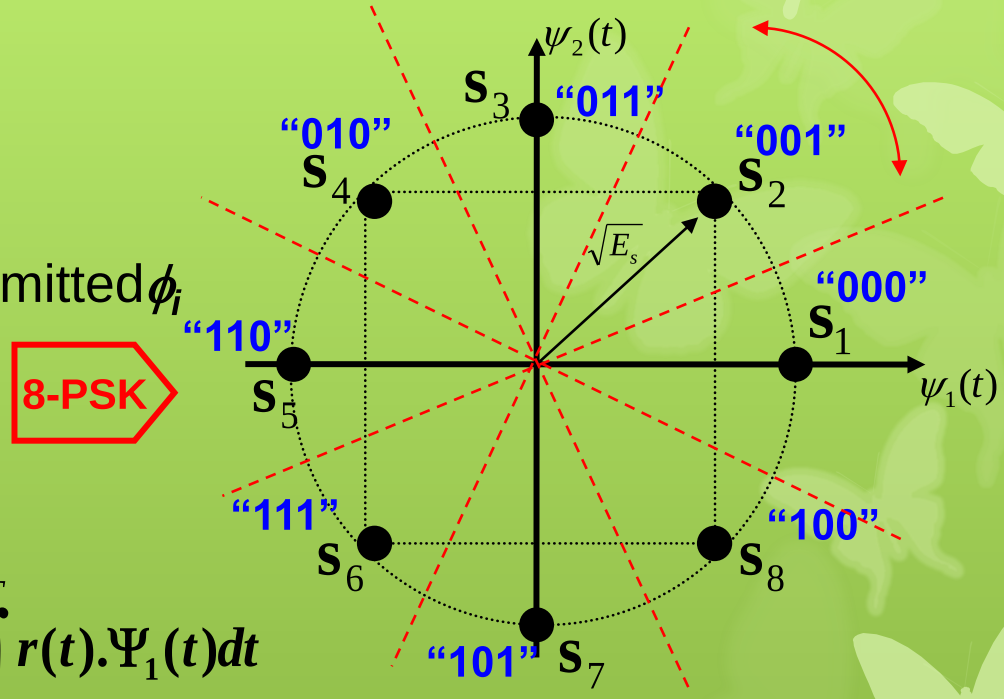
Bandwidth

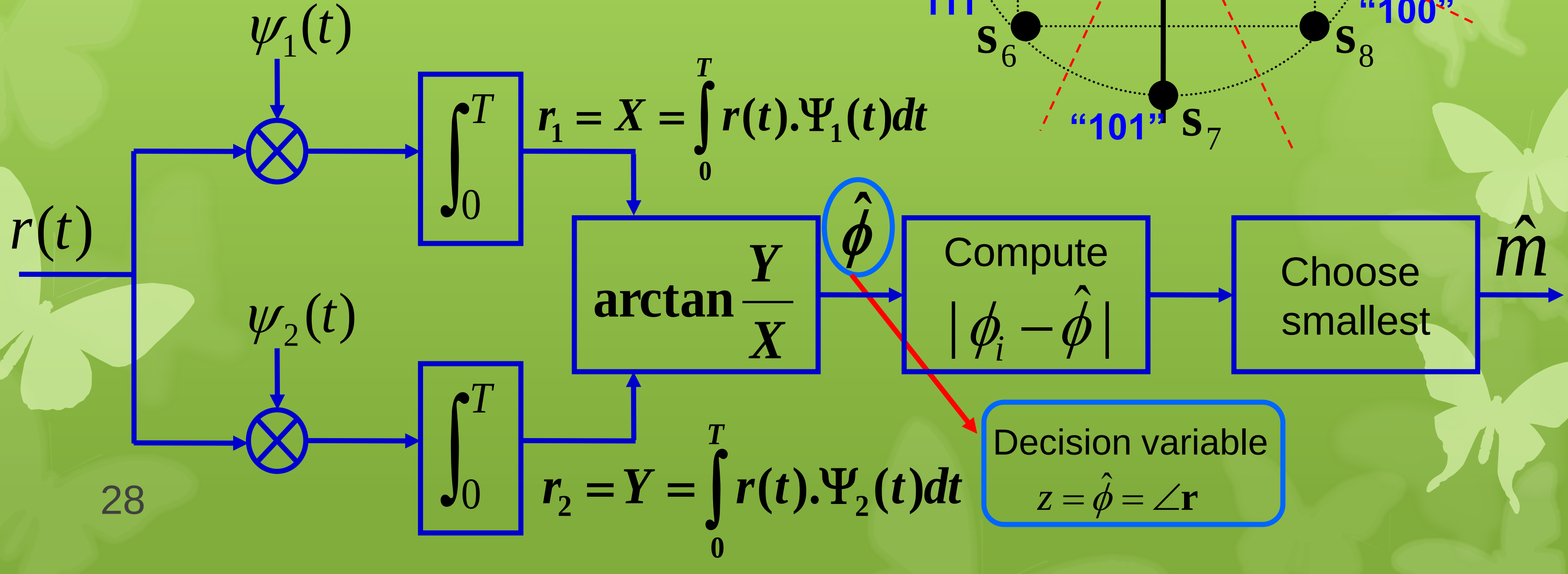
BPSK	QPSK
R_b	$R_b/2$

Error probability …

$\hat{\phi}$ is a noisy estimate of the transmitted ϕ_i

- ***Coherent detection of M-PSK***





Error probability …

- *Coherent* detection of MPSK …
- The detector compares the phase of observation vector to M-1 thresholds.
- Due to the circular symmetry of the signal space, we have:

$$P_E(M) = 1 - P_C(M) = 1 - \frac{1}{M}\sum_{m=1}^{M} P_c(\mathbf{s}_m) = 1 - P_c(\mathbf{s}_1) = 1 - \int_{-\pi/M}^{\pi/M} p_{\hat{\phi}}(\phi)\,d\phi$$

where

$$p_{\hat{\phi}}(\phi) \approx \sqrt{\frac{2}{\pi}\frac{E_s}{N_0}}\cos(\phi)\exp\left(-\frac{E_s}{N_0}\sin^2\phi\right); \quad |\phi| \le \frac{\pi}{2}$$

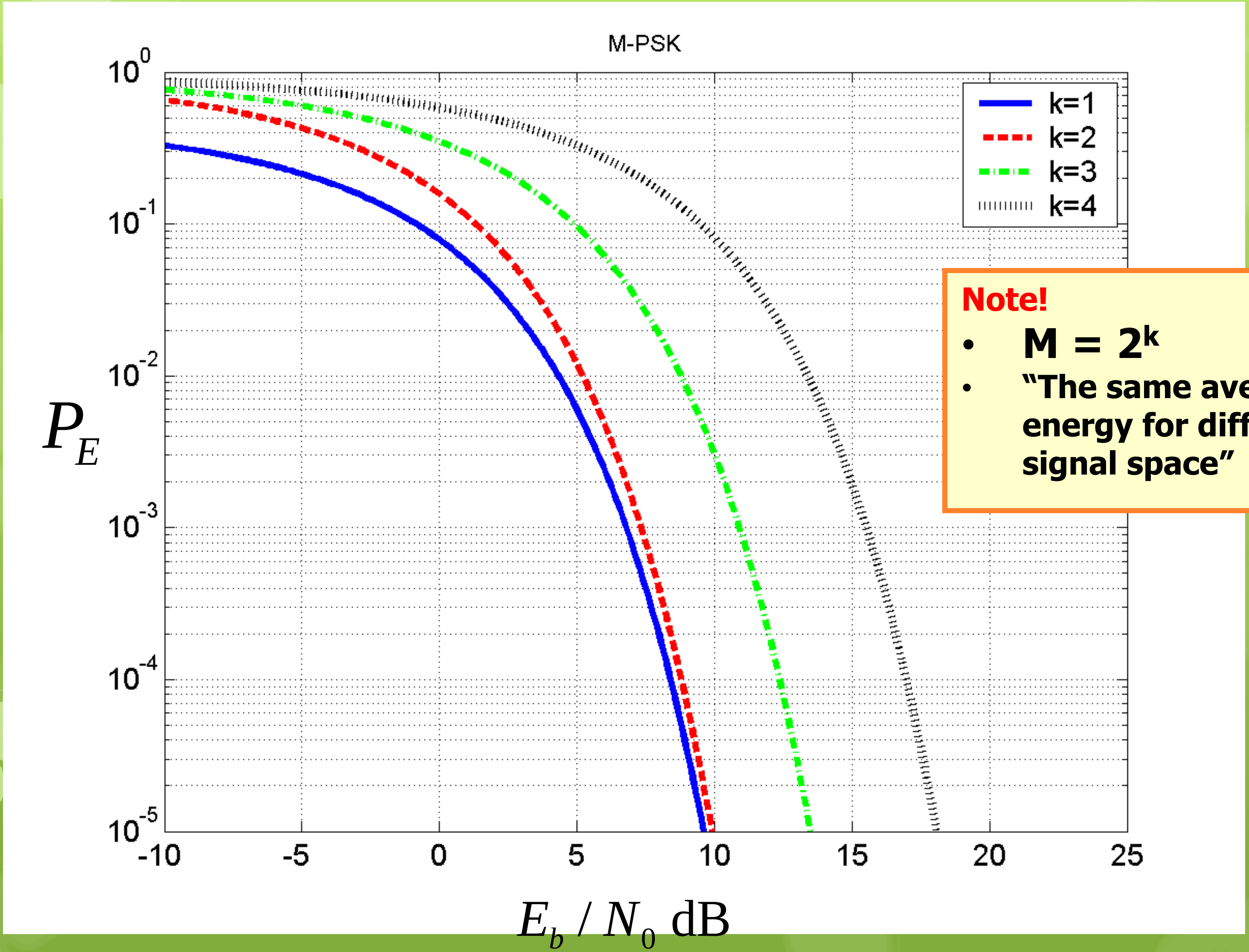
- It can be shown that (**for M > 4**)

$$P_E(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$

or

$$\mathbf{P_E(M) \approx 2Q\left(\sqrt{\frac{2(\log_2 M)E_b}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)}$$

Probability of symbol error for M-PSK



Note!

- **$M = 2^k$**
- **"The same average symbol energy for different sizes of signal space"**

THANK YOU