MODULASI FSK

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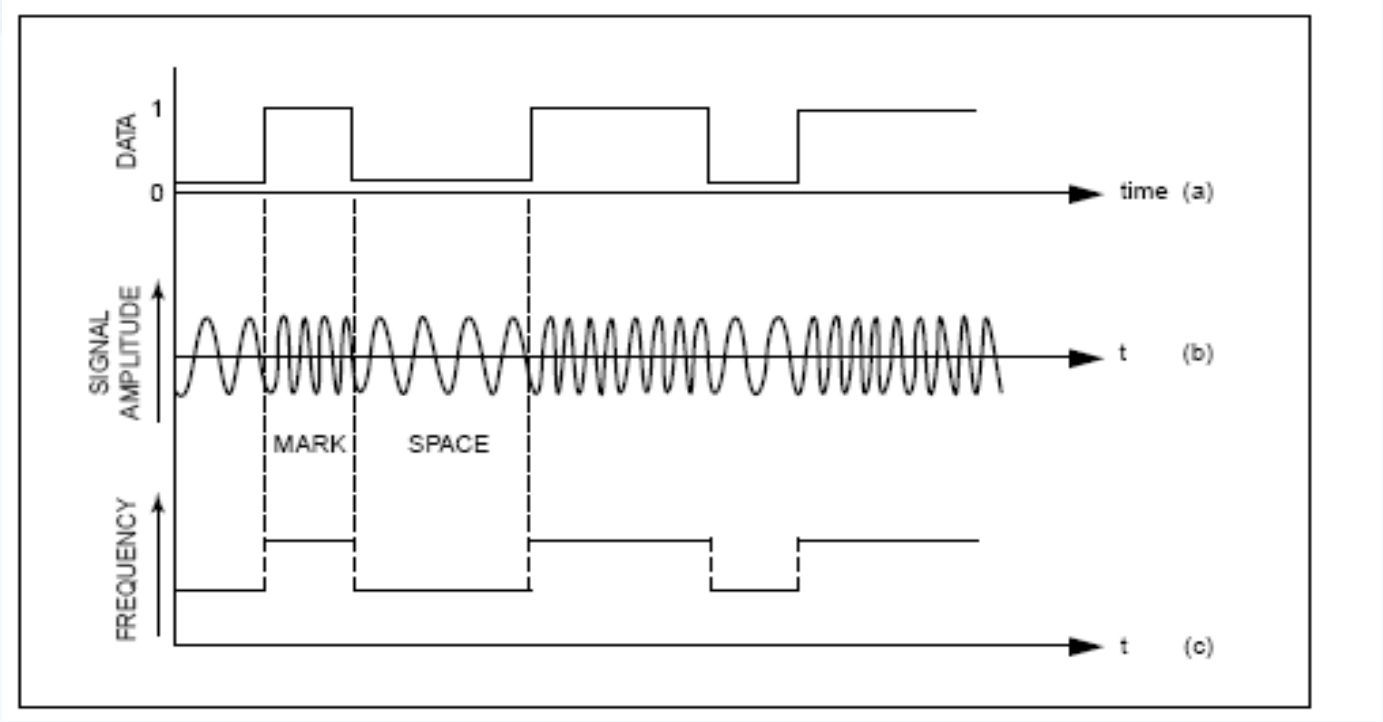
Binary FSK



$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_0 t) \quad 0 \le t \le T_b$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t) \quad 0 \le t \le T_b$$

f_0 and f_1 are chosen such that

$$\int_0^{T_b} \cos(2\pi f_0 t)\cos(2\pi f_1 t)\,dt = 0 \rightarrow \text{orthogonal}$$

Demodulation & Detection M-FSK

- M-ary Frequency Shift Keying (M-FSK)

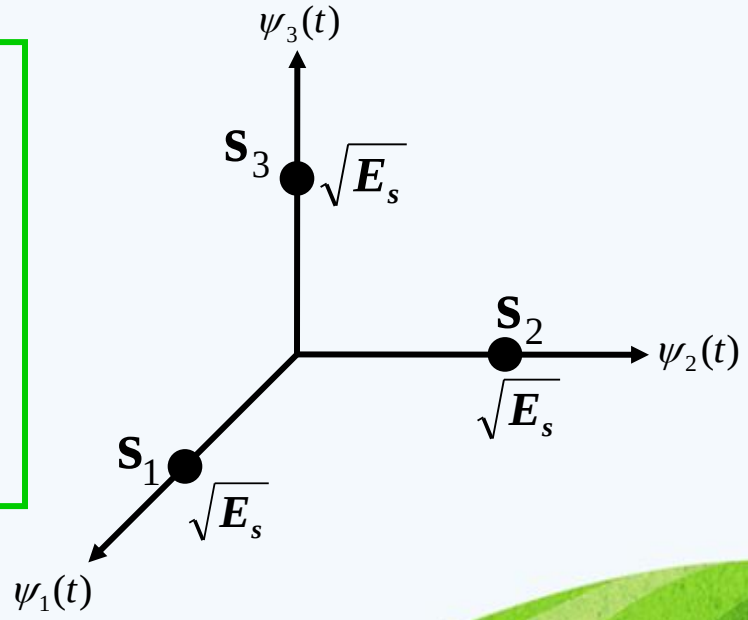
$$s_i(t) = \sqrt{\frac{2E_s}{T}}\cos(\omega_i t) = \sqrt{\frac{2E_s}{T}}\cos(\omega_c t + (i-1)\Delta\omega t)$$

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{1}{2T}$$

$$s_i(t) = \sum_{j=1}^{M} a_{ij}\psi_j(t) \quad i = 1,\ldots,M$$

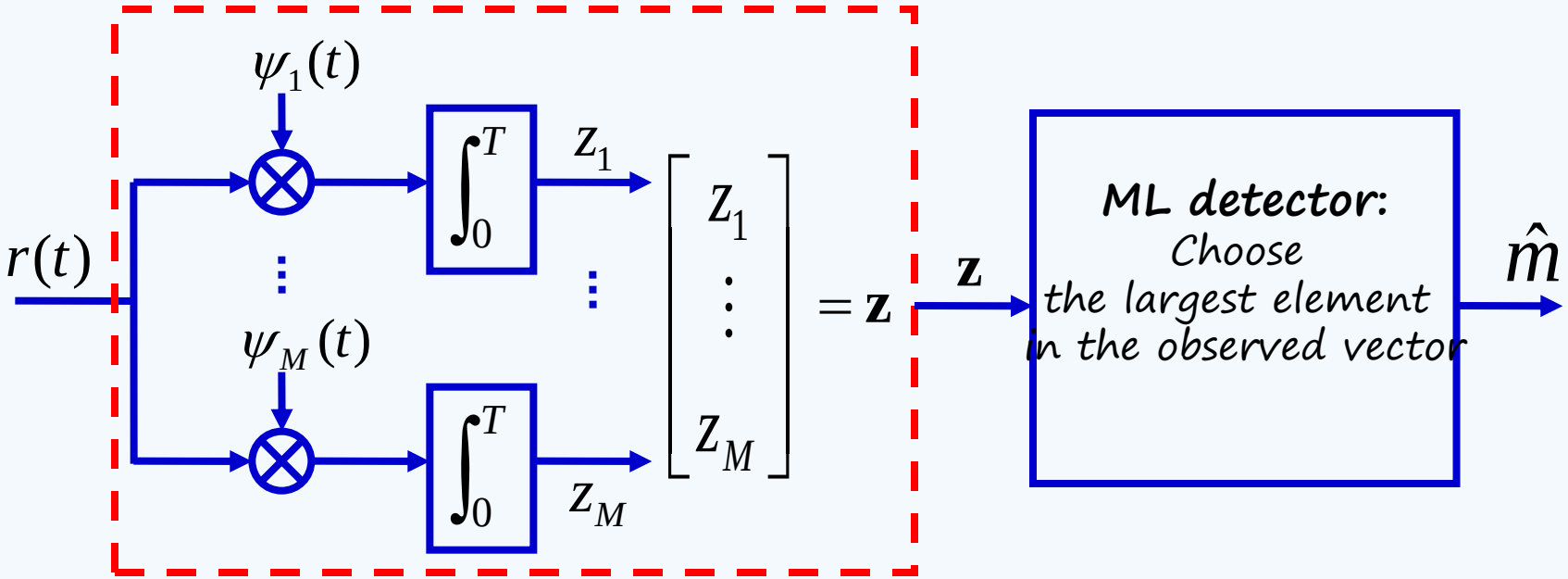
$$\psi_i(t) = \sqrt{\frac{2}{T}}\cos(\omega_i t) \qquad a_{ij} = \begin{cases} \sqrt{E_s} & i = j \\ 0 & i \neq j \end{cases}$$

$$E_s = E_i = \|\mathbf{s}_i\|^2$$



Demodulation & Detection M-FSK

- *Coherent detection of M-FSK*



Error probability …

- Coherent detection of M-FSK …
- The dimensionality of signal space is M. An upper bound for average symbol error probability can be obtained by using union bound. Hence

$$P_E(M) \le (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

or, equivalently

$$P_E(M) \le (M-1)Q\left(\sqrt{\frac{(\log_2 M)E_b}{N_0}}\right)$$

Bit error probability versus symbol error probability

- Number of bits per symbol $k = \log_2 M$
- For orthogonal M-ary signaling (M-FSK)

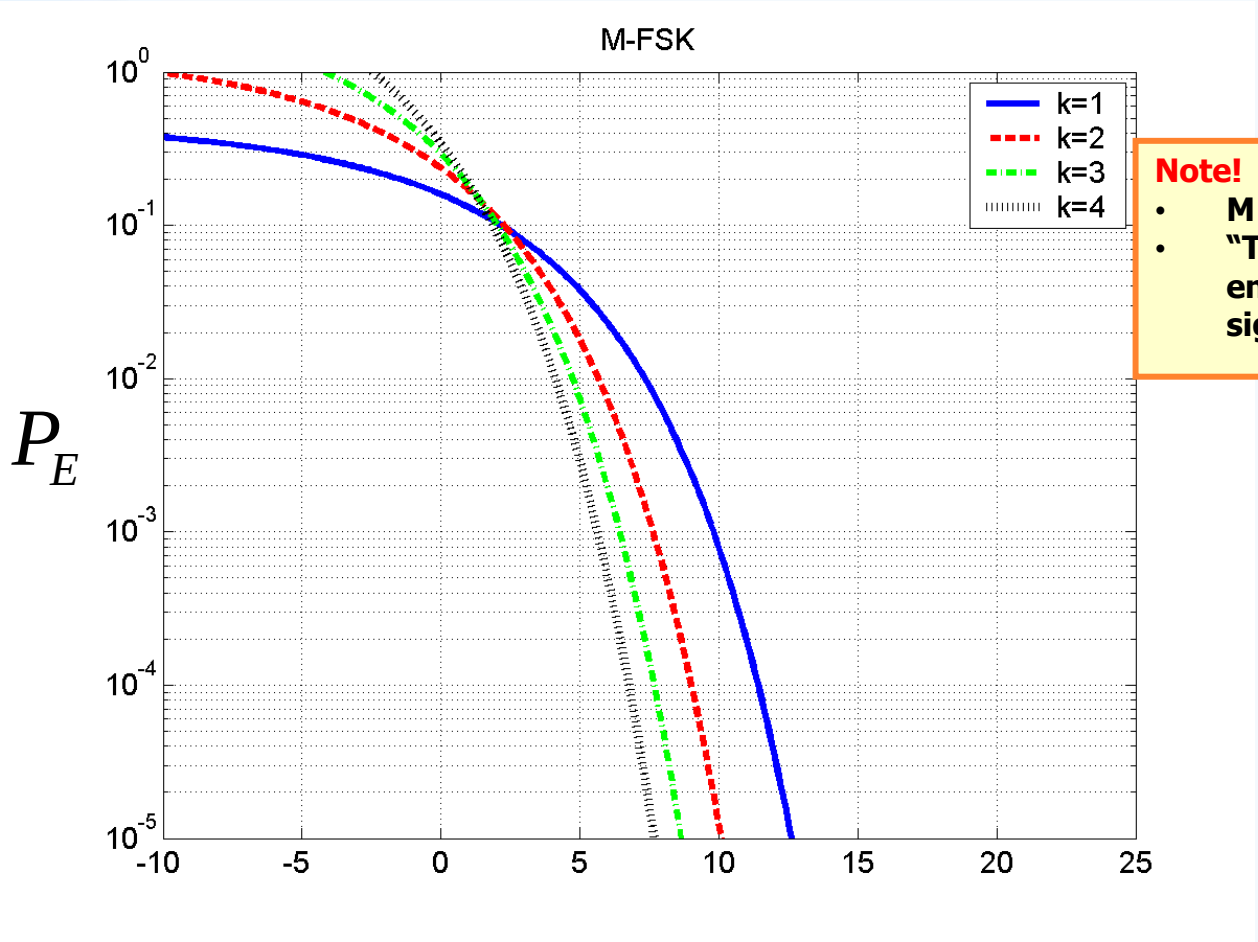
$$\frac{P_B}{P_E} = \frac{2^{k-1}}{2^k - 1} = \frac{M/2}{M-1}$$

$$\lim_{k\to\infty} \frac{P_B}{P_E} = \frac{1}{2}$$

- For M-PSK, M-PAM and M-QAM

$$P_B \approx \frac{P_E}{k} \quad \text{for} \quad P_E << 1$$

Probability of symbol error for M-FSK



Note!

- **M = 2^k**
- **"The same average symbol energy for different sizes of signal space"**

Power Spectral FSK

- The FSK wave can be written as

$$s(t)=\sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t \pm \frac{\pi t}{T_b}\right) \qquad 0 \le t \le T_b$$

$$s(t)=\sqrt{\frac{2E_b}{T_b}}\cos\left(\frac{\pi t}{T_b}\right)\cos(2\pi f_c t) \mp \sqrt{\frac{2E_b}{T_b}}\sin\left(\frac{\pi t}{T_b}\right)\sin(2\pi f_c t)$$

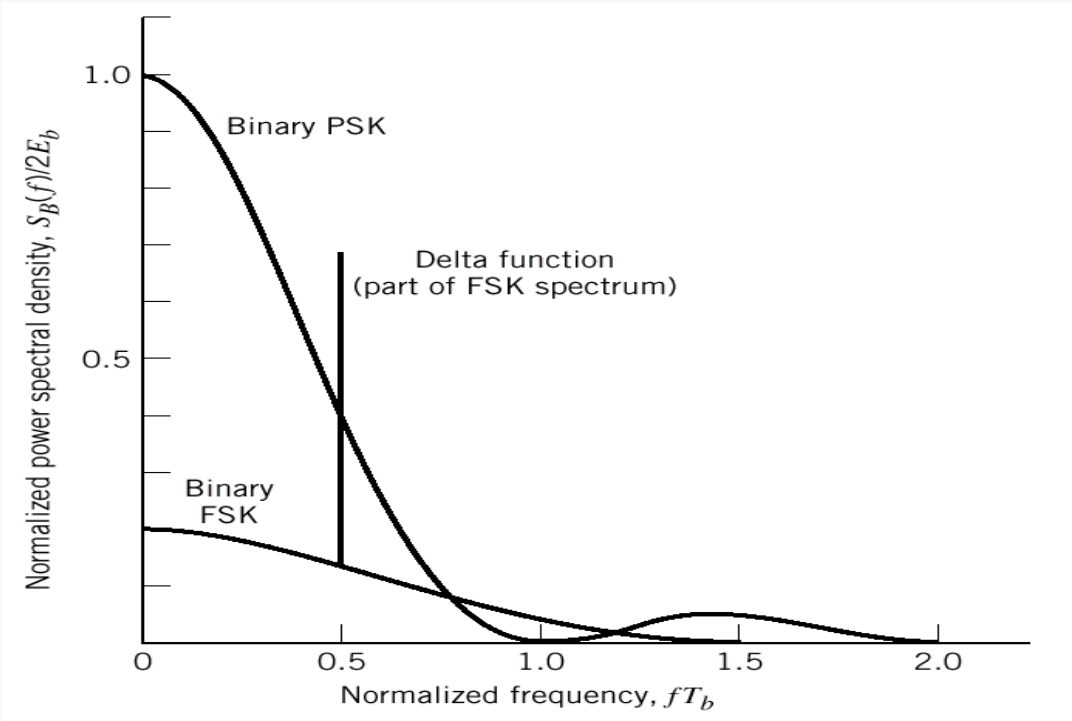
- The in-phase component is completely independent of the input binary wave. The power spectral density of this component consists of two delta functions at f=+- 1/2Tb
- The quadrature component is related to the input binary wave and is given by

$$g(t)=\begin{cases}\sqrt{\dfrac{2E_b}{T_b}} & 0 \le t \le T_b \\ 0 & \text{otherwise}\end{cases}$$

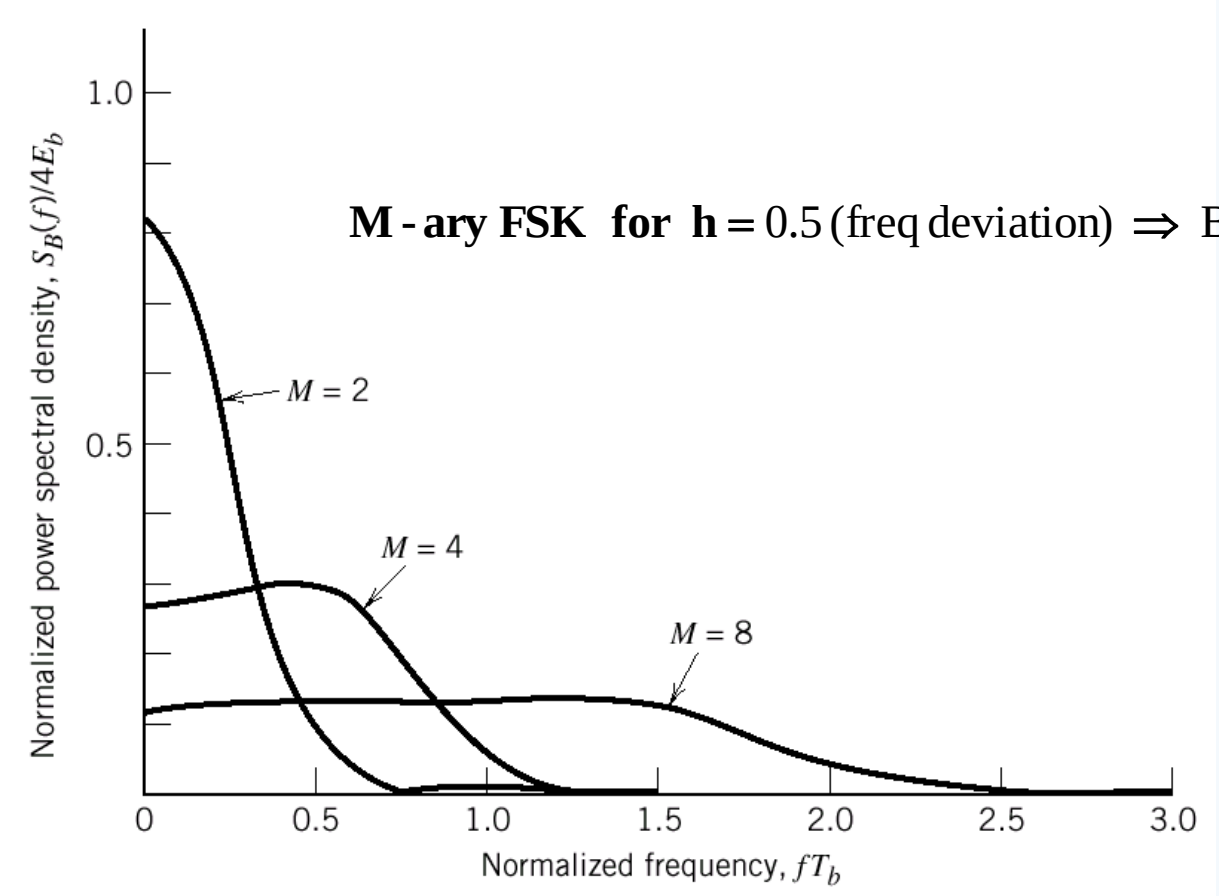
$$\Psi_g(f)=\frac{8E_bT_b\cos^2(\pi T_b f)}{\pi^2(4T_b^2 f^2-1)^2}$$

Power Spectral FSK

$$PSD = \frac{E_b}{2T_b}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right] + \frac{8E_bT_b\cos^2(\pi T_b f)}{\pi^2(4T_b^2 f^2 - 1)^2}$$



Power Spectral M-FSK



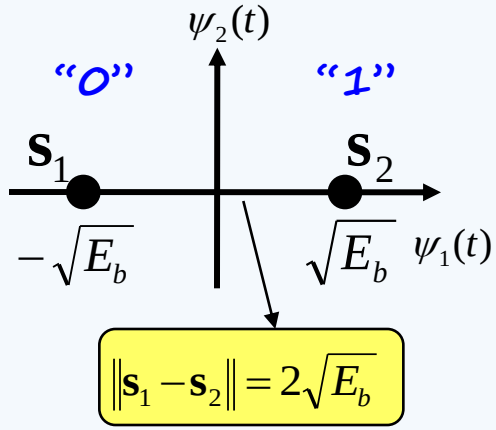
$$\textbf{M-ary FSK for } \mathbf{h} = 0.5\,(\text{freq deviation}) \Rightarrow \text{BW} \approx \log_2 M \bullet \frac{\mathbf{0.5}}{T_b}$$

Let's compare Error probability: BPSK vs BFSK

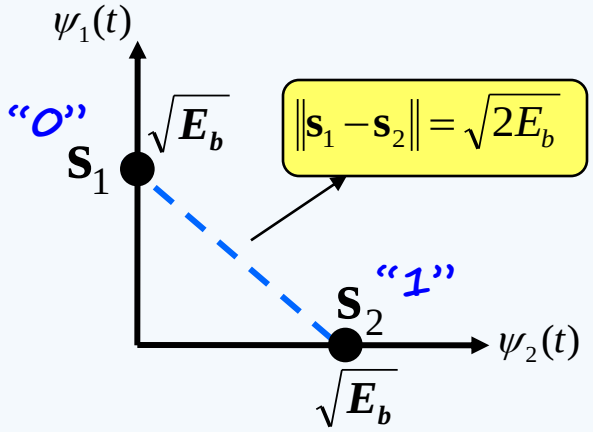
- BPSK and BFSK with ***coherent*** detection:

$$P_B = Q\left(\frac{\|\mathbf{s}_1 - \mathbf{s}_2\|/2}{\sqrt{N_0/2}}\right)$$

BPSK



BFSK



$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

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Contoh Soal

- Suatu sinyal 2-FSK mempunyai amplituda ± 100 m volt, frekuensi pembawa (center) = 100 MHz, deviasi frekuensi = Rb/2, Bit Rate Informasi = Rb = 1 Mbps.
  - Gambarkan gelombang FSK tsb dan Gambarkan konstellasi sinyalnya!
  - Gambarkan spektrum frekuensinya !
  - Berapakah Bandwidth sinyal FSK tsb jika roll of factor α = 0.3 !
  - Berapakah Pe (BER) jika sinyal FSK tsb bercampur dengan noise AWGN No= 10^{-10} W/Hz

THANK U