TRANSFORMASI FOURIER

SISTEM KOMUNIKASI (DTG2F3)

PRODI D3 TT

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FUNGSI DAN DEFINISI

- Spektral sinyal periodik s(t) selalu dapat dianalisis dengan bantuan Deret Fourier.
- Pada kenyataannya banyak sinyal-sinyal dalam sistem komunikasi yang bersifat random non periodik, misalnya sinyal informasi.
- Untuk kasus sinyal non periodik kita gunakan formula yang disebut Transformasi Fourier.
- Fungsi Transformasi Fourier yaitu utk menganalisis bentuk spektral S(f) dari suatu sinyal kawasan waktu s(t)
- Fungsi Inverse Transformasi Fourier yaitu utk menganalisis bentuk suatu sinyal kawasan waktu s(t) jika spektral sinyal S(f) diketahui

Formula Transformasi Fourier

$$S(f) = \int_{-\infty}^{+\infty} s(t).e^{-j2\pi ft}\,dt$$

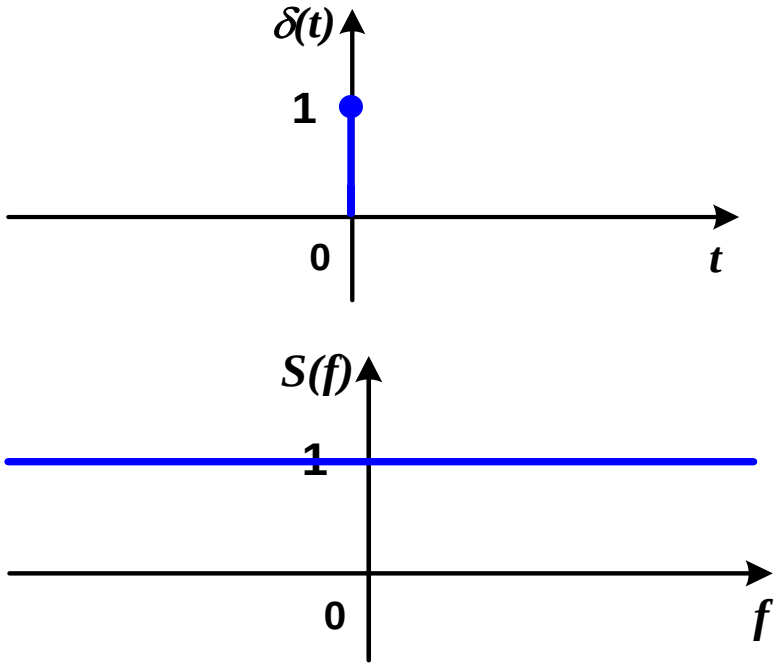
- *S(f)* dinamakan Transformasi Fourier dari s(t)
- Jika Transformasi Fourier *S(f)* suatu sinyal diketahui maka kita dapat menghitung persamaan sinyal dalam domain waktu s(t) dengan formula Inverse Transformasi Fourier

$$s(t) = \int_{-\infty}^{+\infty} S(f).e^{j2\pi ft}\,df$$

Beberapa Transformasi penting

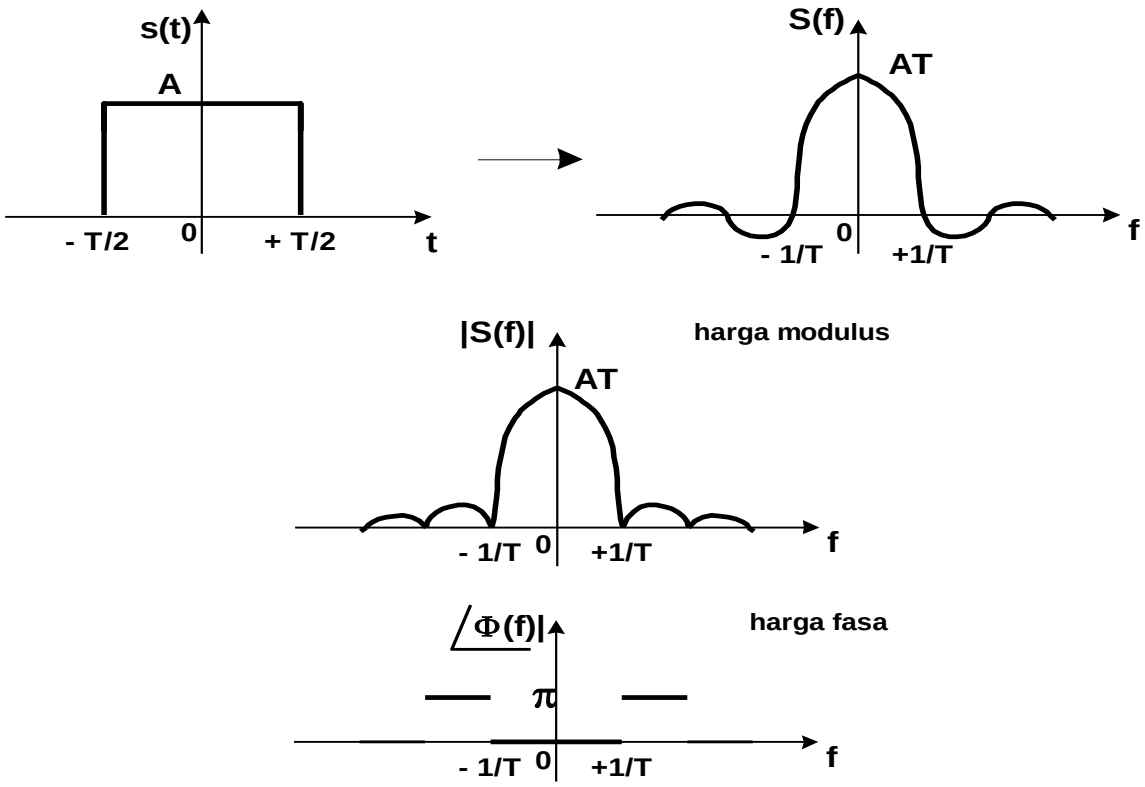
$$S(f) = \int_{-\infty}^{+\infty} \delta(t).e^{-j2\pi ft} dt = 1$$

- Transformasi Fourier impulse (sinyal delta dirac):



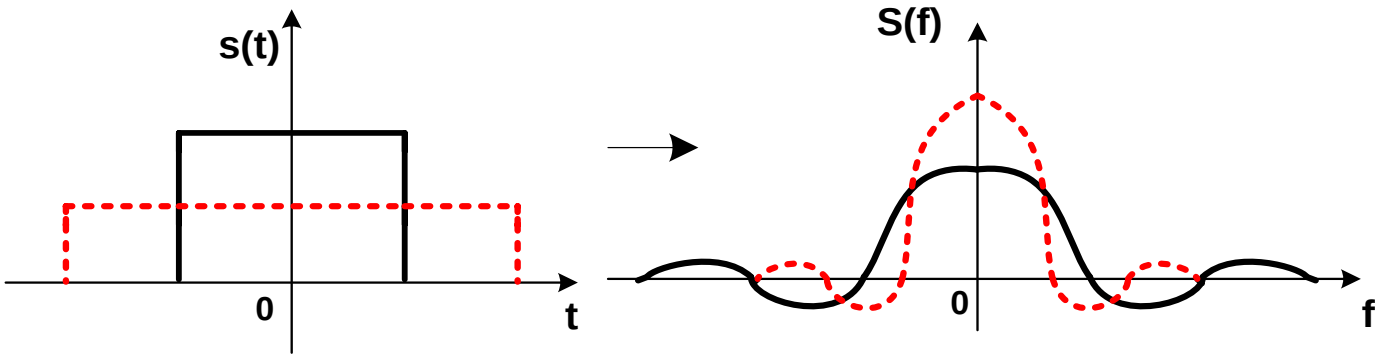
Beberapa Transformasi penting

- Transformasi Fourier dari fungsi pulsa:



Sifat-sifat Transformasi Fourier (yang sering dipakai di siskom)

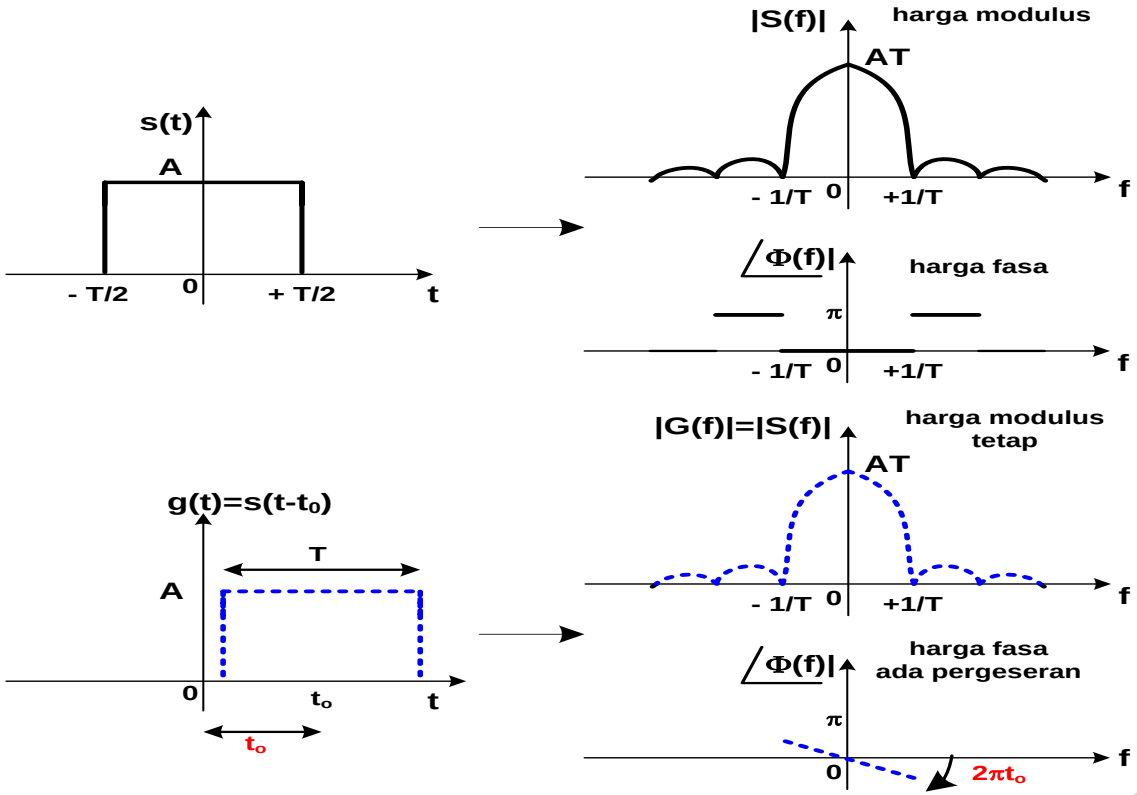
a. Time Scaling



Sifat-sifat Transformasi Fourier

b. Time shifting

Bila $s(t) \leftrightarrow S(f)$ maka $s(t-t_o) \leftrightarrow S(f).e^{-j2\pi fto}$



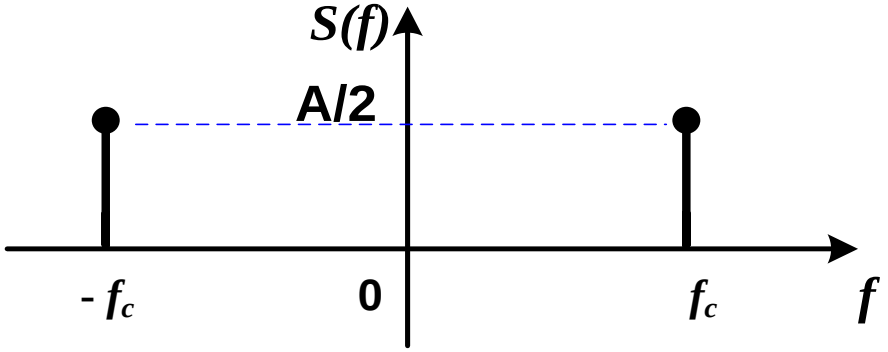
Sifat-sifat Transformasi Fourier

c. Frequency shifting

Bila $s(t) \leftrightarrow S(f)$ maka $S(f-f_o) \leftrightarrow s(t).e^{j2\pi f_o t}$

- Contoh : s(t) = A Cos $2\pi f_c t$ = $\frac{A}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)$
- maka

$$S(f) = \frac{A}{2}\delta(f + f_c) + \frac{A}{2}\delta(f - f_c)$$



Sifat-sifat Transformasi Fourier

d. Transformasi Fourier Sinyal Periodik

Bila $x(t) \leftrightarrow X(f)$ (untuk sinyal tidak periodik)

Maka untuk $$x_p(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0)$$

(➔x(t) periodik dengan periode T_o)

Transformasi fourier dari $x_p(t)$

$$X_p(f) = \frac{1}{T_0} \sum_{m=-\infty}^{+\infty} X\left(\frac{m}{T_o}\right).\delta\left(f - \frac{m}{T_0}\right)$$

Sifat-sifat Transformasi Fourier

e. Integrasi pada kawasan waktu:

Bila $s(t) \leftrightarrow S(f)$, kemudian menghasilkan $S(f)=0$,

maka : $$\int_{-\infty}^{t} s(t).dt \Leftrightarrow \frac{1}{j2\pi f}.S(f)$$

f. Diferensiasi pada kawasan waktu:

Bila $s(t) \leftrightarrow S(f)$, jika pada kawasan waktu dilakukan diferensiasi sekali, *maka :*

$$\frac{d}{dt}s(t) \Leftrightarrow j2\pi f.S(f)$$

Sifat-sifat Transformasi Fourier

g. Konvolusi pada kawasan waktu:

Bila $s_1(t) \leftrightarrow S_1(f)$ dan $s_2(t) \leftrightarrow S_2(f)$,

maka :

$$\int_{-\infty}^{\infty} s_1(\tau).s_2(t-\tau)d\tau \Leftrightarrow S_1(f).S_2(f)$$

h. Perkalian pada kawasan waktu:

Bila $s_1(t) \leftrightarrow S_1(f)$ dan $s_2(t) \leftrightarrow S_2(f)$,

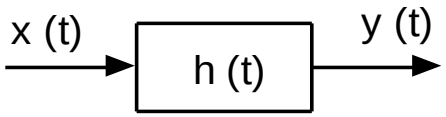
maka :

$$s_1(t).s_2(t) \Leftrightarrow \int_{-\infty}^{\infty} S_1(\lambda).S_2(f-\lambda)d\lambda$$

Transmisi Sinyal melalui Sistem Linier

Respon Time :

Time Domain



h (t) ≡ respon impuls

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

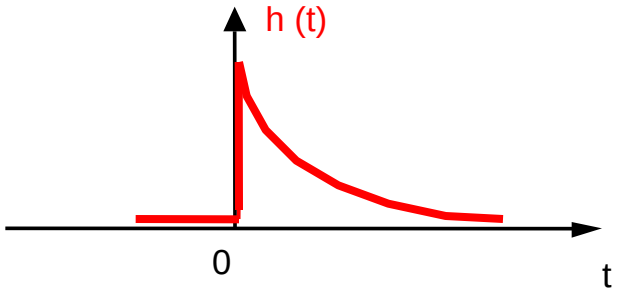
$$= \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

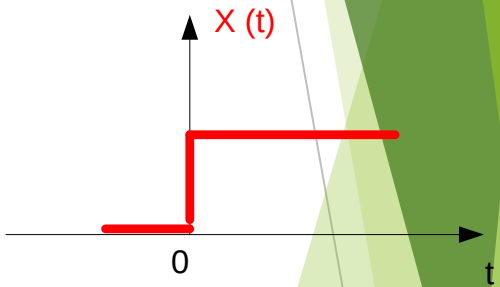
$$= x(t) \otimes h(t)$$

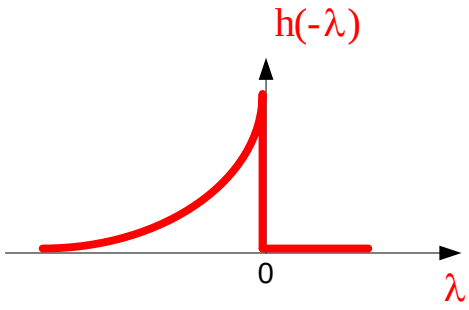
$$= h(t) \otimes x(t)$$

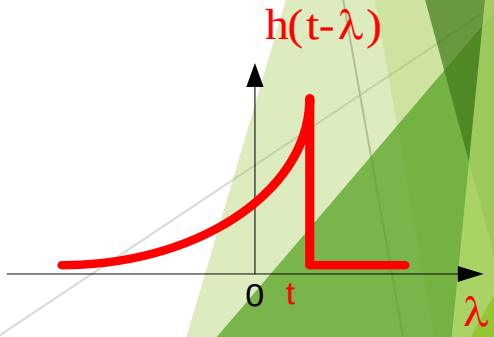
Perhitungan Konvolusi :

Representasi Grafis ; contoh

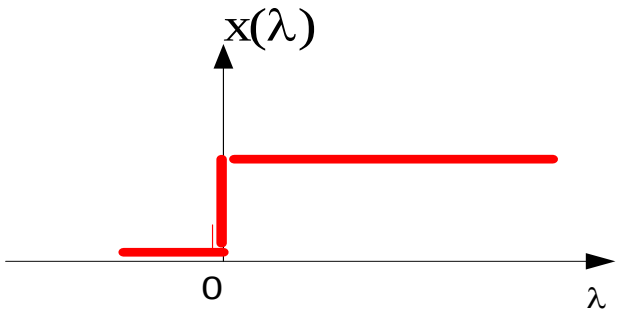


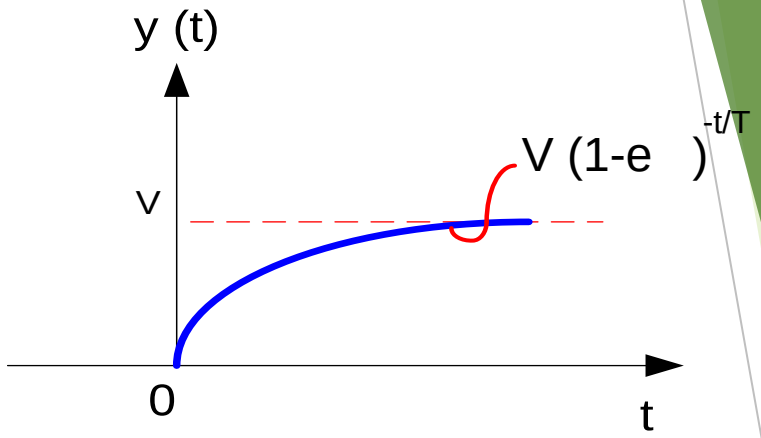




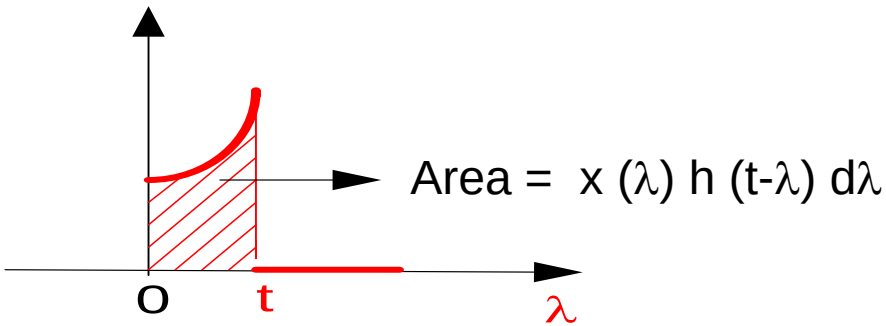


Transmisi Sinyal melalui Sistem Linier(CONT')

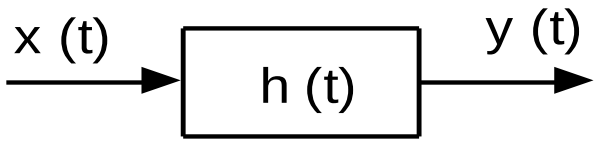


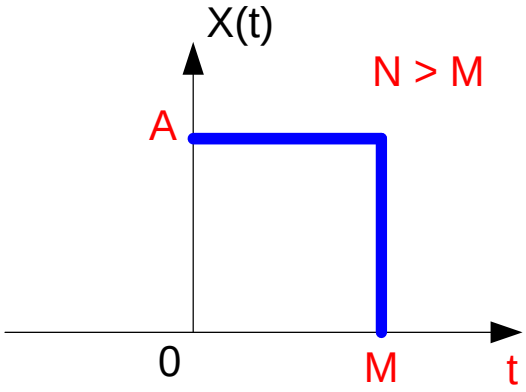


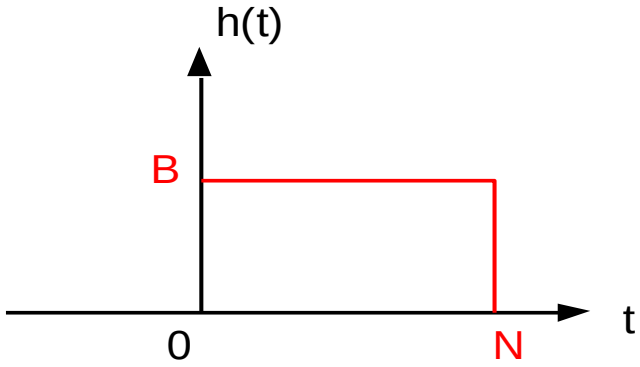
$$\int_0^t x(\lambda)\, h(t-\lambda)$$

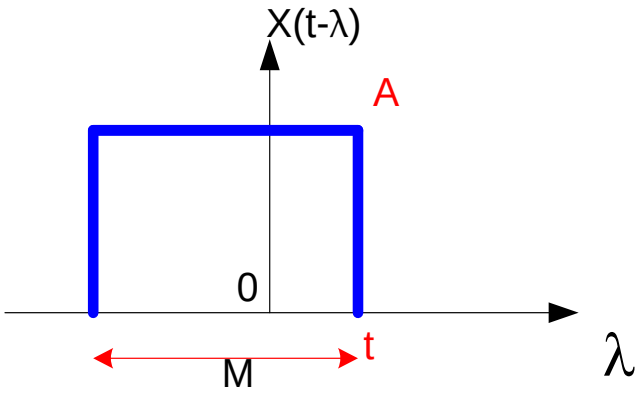


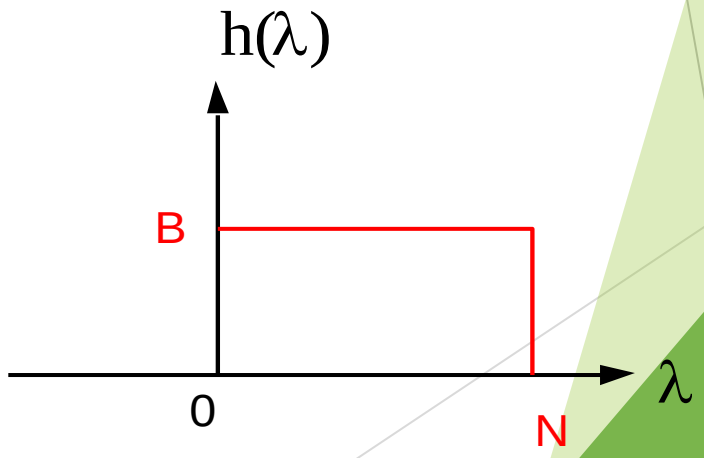
Contoh Perhitungan Konvolusi dgn representasi Grafis :



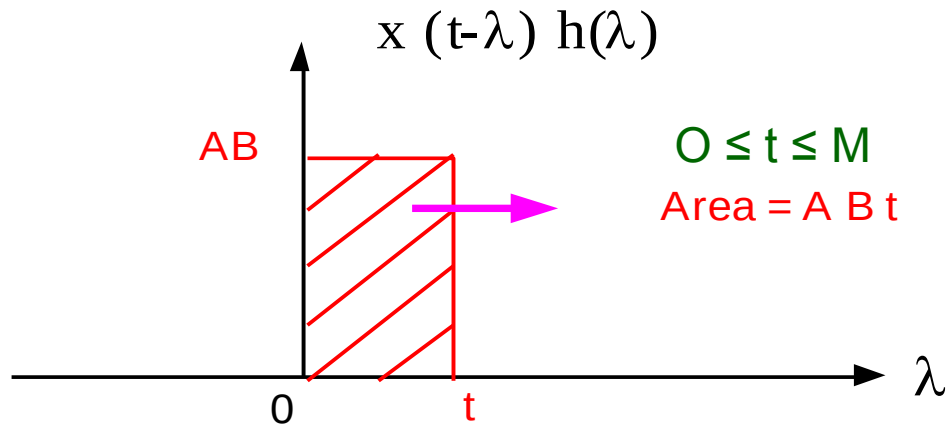








Transmisi Sinyal melalui Sistem Linier (CONT')

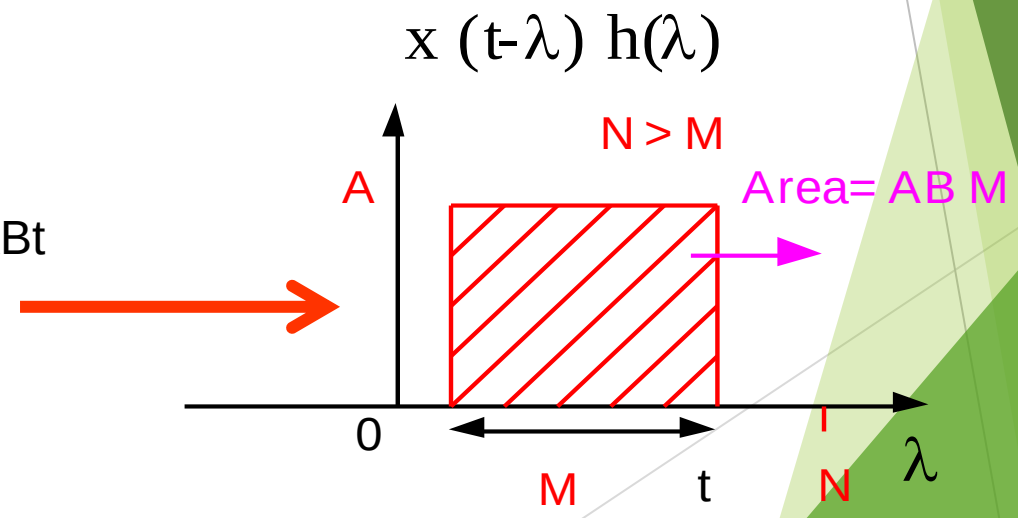


Perhitungan

Karena N > M :

untuk 0 ≤ t ≤ M : y(t) = ABt

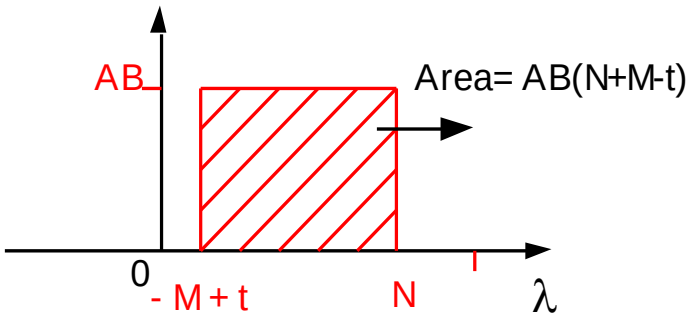
untuk M ≤ t ≤ N :



Transmisi Sinyal melalui Sistem Linier (CONT;)

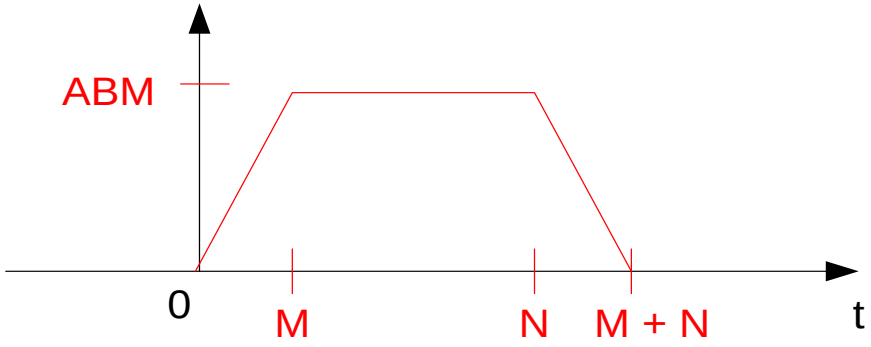
untuk t ≥ N :

$$x(t-\lambda)\,h(\lambda)$$



Sehingga:

$$y(t)=x(t)\otimes h(t)$$

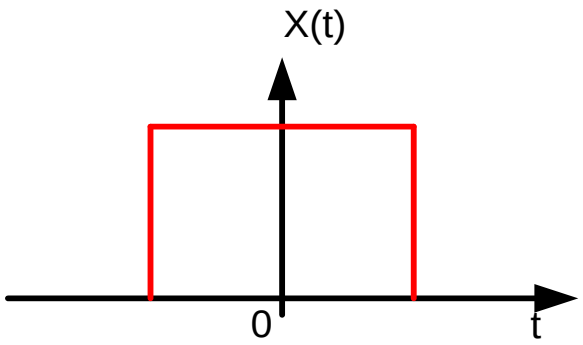


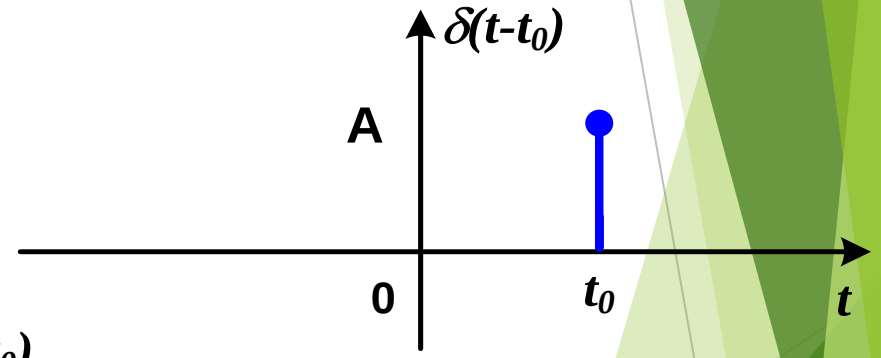
Kasus Khusus :

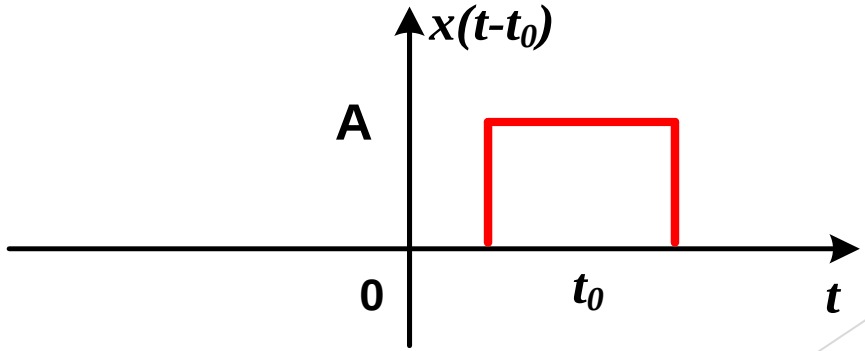
Konvolusi dengan fungsi δ (t - to)

- $x(t) \otimes \delta(t - to) = \int_{-\infty}^{\infty} x(t - \lambda)\, \delta(\lambda - to)\, d\lambda = x(t - to)$

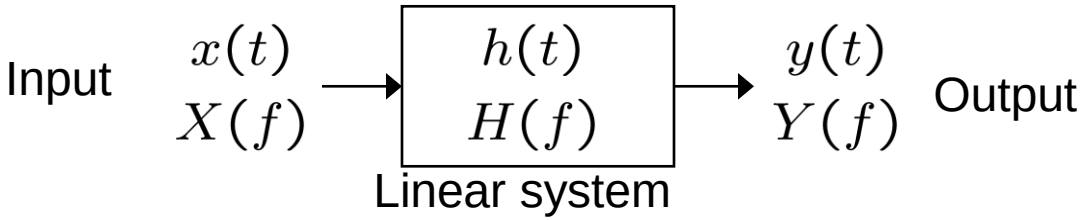
- $x(t) \otimes A\,\delta(t - to) = A\,x(t - to)$







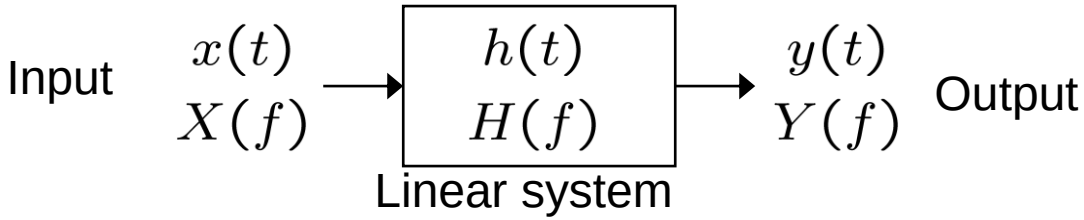
Transmisi Sinyal Melalui Sistem Linier



- Deterministic signals: $Y(f) = X(f)H(f)$
- Random signals: $G_Y(f) = G_X(f)|H(f)|^2$

- Y(f) = Sinyal output dalam domain frekuensi
- X(f) = Sinyal input dalam domain frekuensi
- H(f) = Respons frekuensi sistem linier
- $G_Y(f)$ = PSD (Power Spectral Density) sinyal output
- $G_X(f)$ = PSD (Power Spectral Density) sinyal input

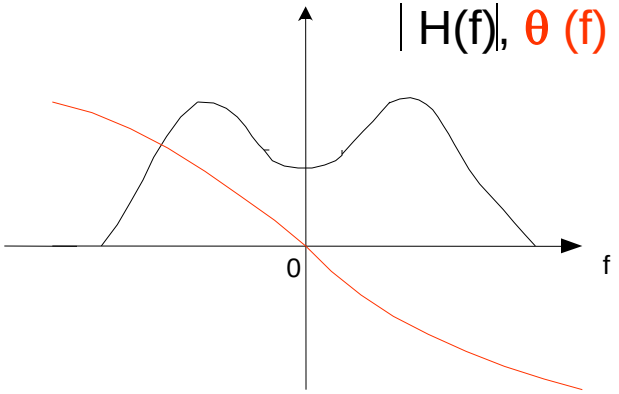
Sistem Lowpass vs Bandpass



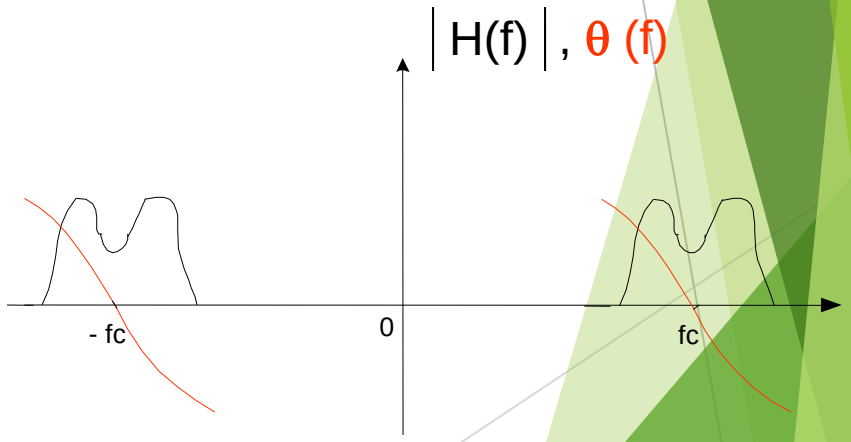
Jika h (t) riil $\Rightarrow$ H (f) kompleks $\rightarrow$ | H (f) | merupakan fungsi genap

$\rightarrow$ θ (f) merupakan fungsi ganjil

Sistem "lowpass"



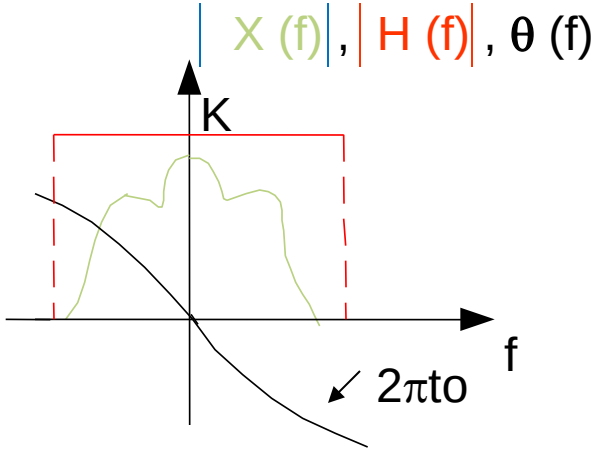
Sistem "bandpass"



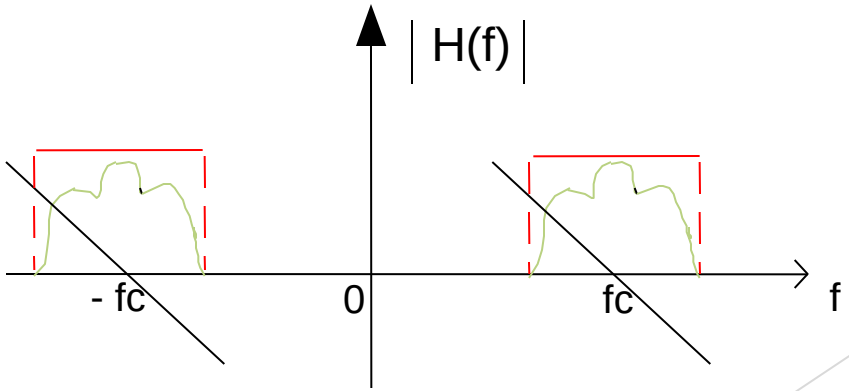
• Kondisi "distortionless transmission"

$y(t) = K.X(t - to)$

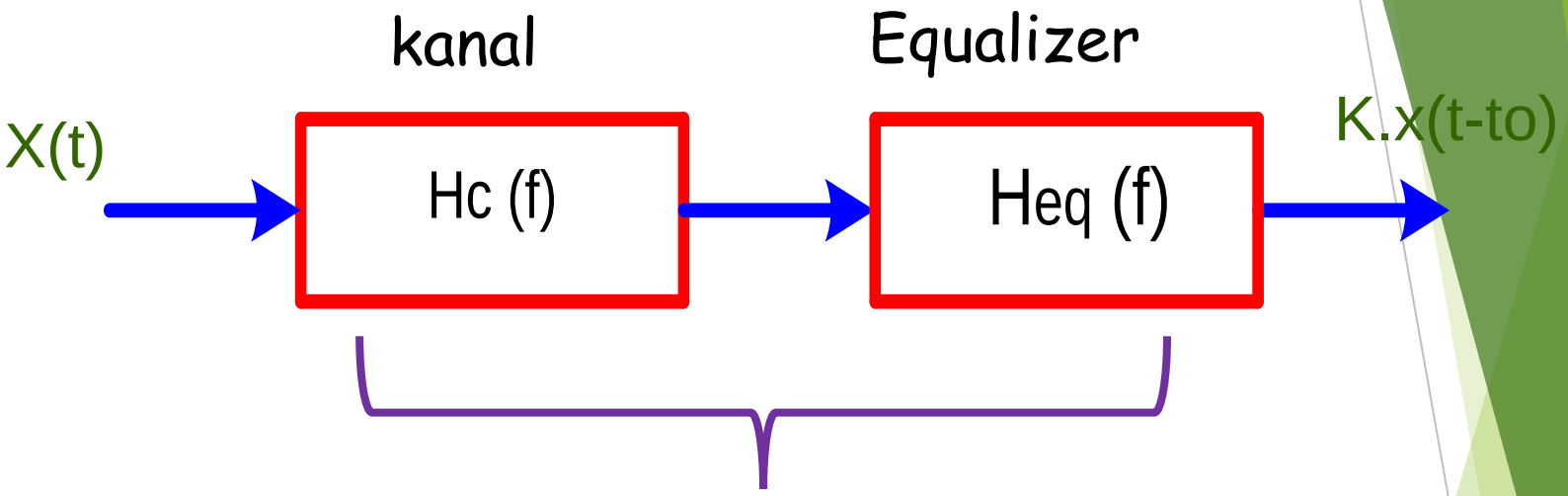
$H(f) = K\, e^{-j2\pi fto}$



• Untuk sistem "bandpass"



• Distorsi Linier dan Prinsip Ekualisasi Kanal

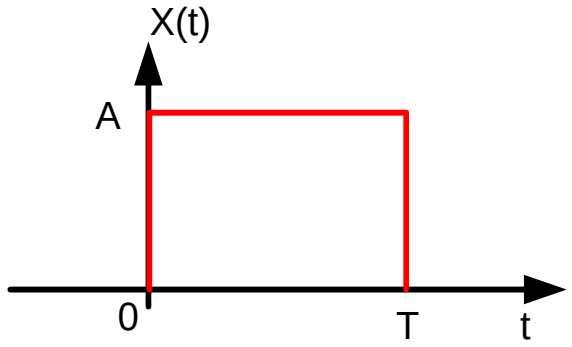


$$Hc(f)\ Heq(f) = K\ e^{-j2\pi fto}$$

$$Heq(f) = \frac{K\ e^{-j2\pi fto}}{Hc(f)}$$

Latihan Soal

1. Perhatian gambar sinyal ***x(t)*** diawah ini :



a. Tentukan ***X(f)*** yang merupakan transformasi fourier dari sinyal tersebut !

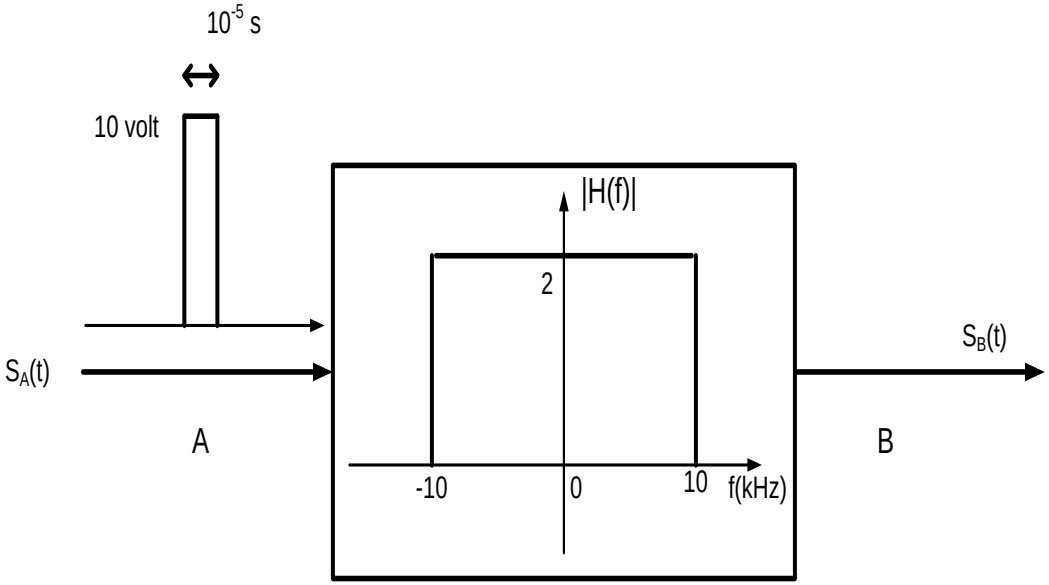
b. Jika sinyal ***z(t)= x(t).y(t)***

   dimana ***y(t) = Cos (4π t/T)***, tentukan ***Z(f)*** !

c. Gambarkan z(t) dan Z(f)

Latihan Soal

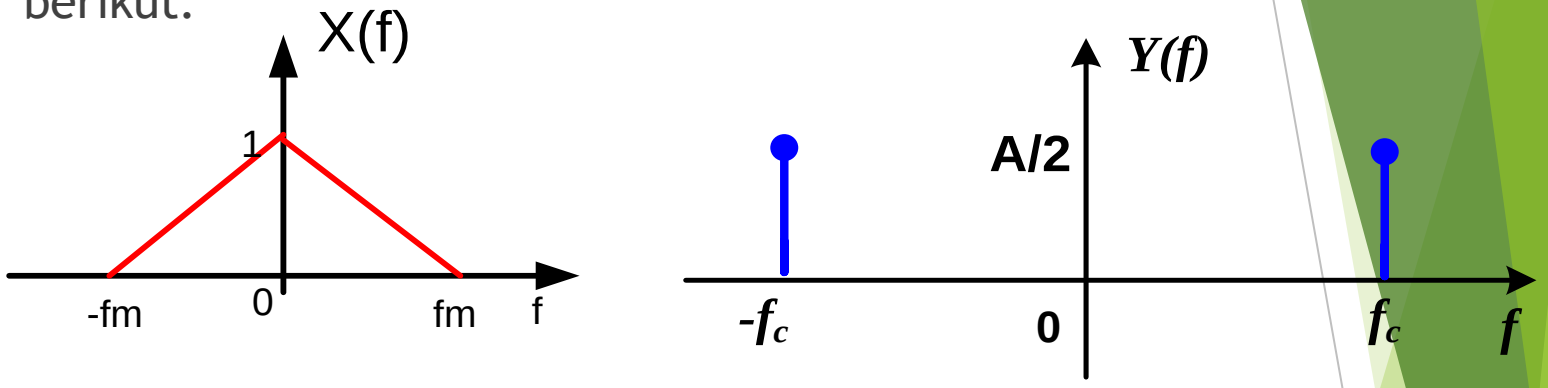
2. Suatu sinyal memasuki sistem yang diwakili oleh LPF berikut ini :



Tentukan $S_A(f)$, $S_B(f)$, $S_B(t)$!

Latihan Soal

3. Diketahui sinyal dalam domain frekuensi sebagai berikut:



a. Untuk fc > fm, Gambarkan Z(f) = X(f)∗Y(f) !

b. Tentukan persamaan z(t), gambar diagram proses yang terjadi !

THANK U