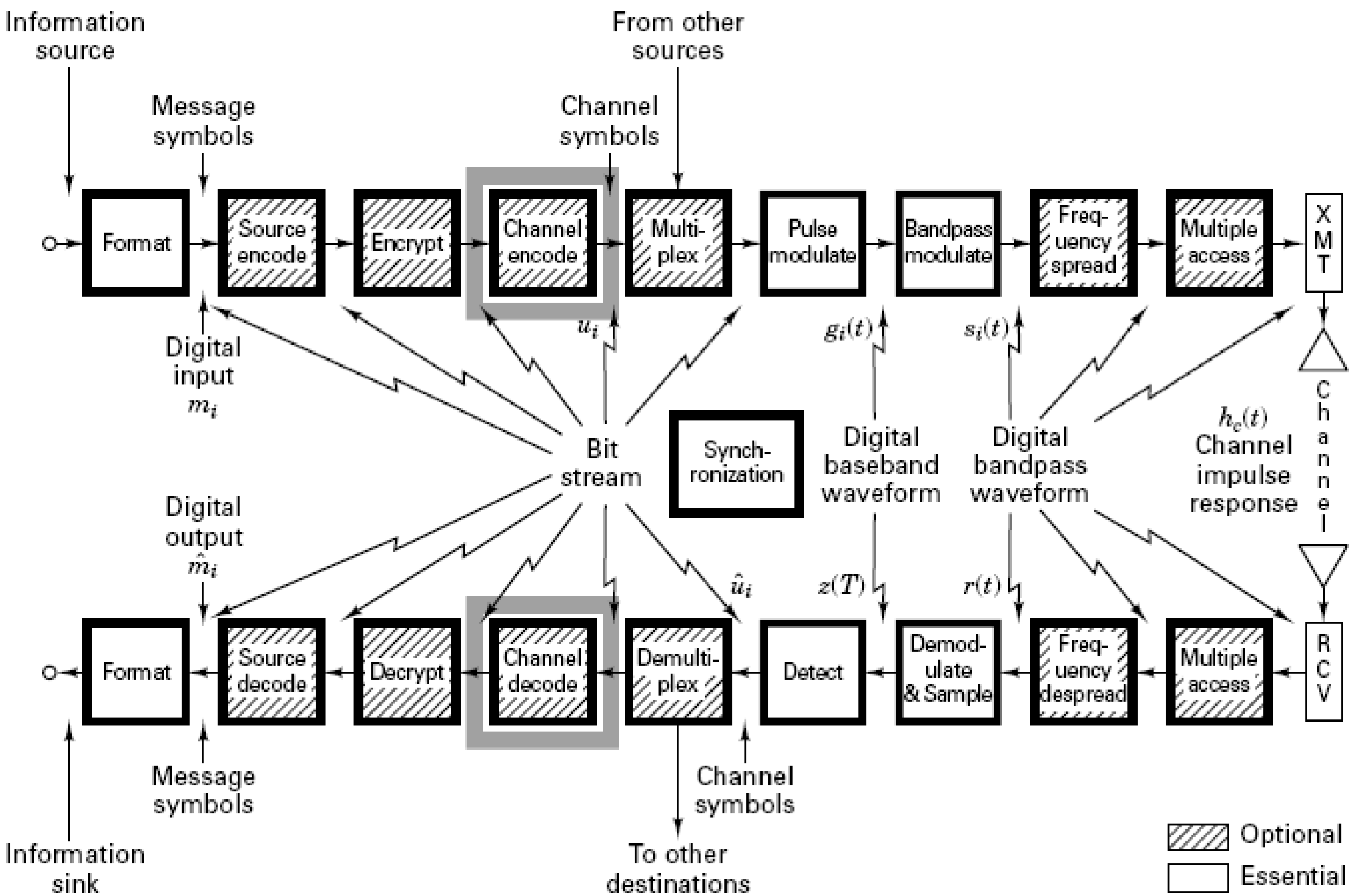
SISTEM KOMUNIKASI

LINEAR BLOCK CODE

Program Studi D3 Teknik Telekomunikasi
FAKULTAS ILMU TERAPAN

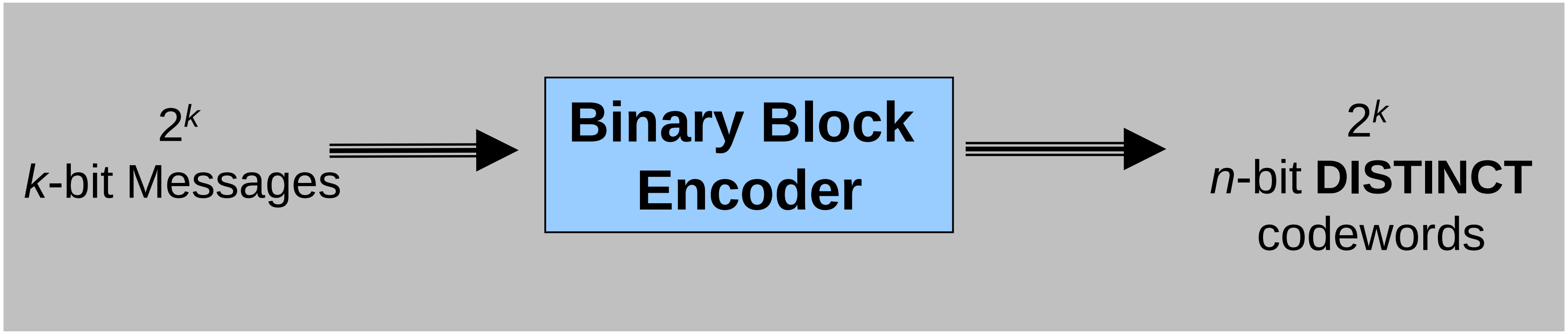
Letak Channel Code



What are Linear Block Codes?

Linear Block Codes

- Information sequence is segmented into message blocks of ***fixed length***.
- Each k-bit information **message** is encoded into an n-bit **codeword** ($n>k$)



What are Linear Block Codes?

Linear Block Codes

- Modulo-2 sum of any two codewords is ……… also a codeword
- Each codeword **v** that belongs to a block code **C** is a linear combination of *k* linearly independent codewords in **C,** i.e.,

$$U = m_0 . g_0 + m_1 . g_1 + \ldots + m_{k-1} . g_{k-1}$$

$$g_i = [g_{i0} \; g_{i1} \; \ldots . \; g_{i,n-1}]$$

Some definitions

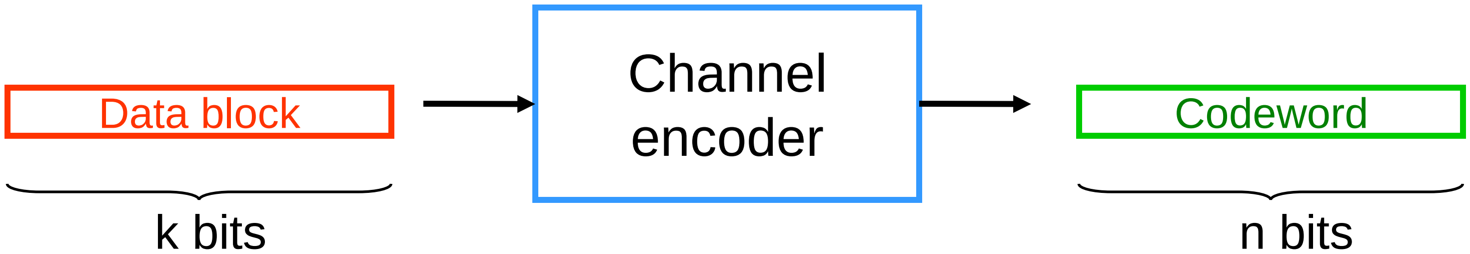
- Binary field :
  - The set {0,1}, under modulo 2 binary addition and multiplication forms a field.

Addition	Multiplication
$0 \oplus 0 = 0$	$0 \cdot 0 = 0$
$0 \oplus 1 = 1$	$0 \cdot 1 = 0$
$1 \oplus 0 = 1$	$1 \cdot 0 = 0$
$1 \oplus 1 = 0$	$1 \cdot 1 = 1$

  - Binary field is also called Galois field, GF(2).

Linear block codes – cont'd

- The information bit stream is chopped into blocks of k bits.
- Each block is encoded to a larger block of n bits.
- The coded bits are modulated and sent over channel.
- The reverse procedure is done at the receiver.



$n\text{-}k$ Redundant bits

$R_c = \dfrac{k}{n}$ Code rate

Linear block codes – cont'd

- The Hamming weight of vector **U**, denoted by w(**U**), is the number of non-zero elements in **U**.
- The Hamming distance between two vectors **U** and **V**, is the number of elements in which they differ.

$$d(\mathbf{U},\mathbf{V}) = w(\mathbf{U} \oplus \mathbf{V})$$

- The minimum distance of a block code is

$$d_{\min} = \min_{i \neq j} d(\mathbf{U}_i, \mathbf{U}_j) = \min_i w(\mathbf{U}_i)$$

Linear block codes – cont'd

- Error detection capability is given by

$$e = d_{\min} - 1$$

- Error correcting-capability **t** of a code, which is defined as the maximum number of guaranteed correctable errors per codeword, is

$$t = \left\lfloor \frac{d_{\min} - 1}{2} \right\rfloor$$

Linear block codes – cont'd

- Encoding in (n,k) block code

$$\mathbf{U} = \mathbf{m}\mathbf{G}$$

$$(u_1, u_2, \ldots, u_n) = (m_1, m_2, \ldots, m_k) \cdot \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \vdots \\ \mathbf{V}_k \end{bmatrix}$$

$$(u_1, u_2, \ldots, u_n) = m_1 \cdot \mathbf{V}_1 + m_2 \cdot \mathbf{V}_2 + \ldots + m_2 \cdot \mathbf{V}_k$$

- The rows of G, are linearly independent.

Linear block codes – cont'd

- Example: Block code (n,k)=(6,3)

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} = \begin{bmatrix} 110100 \\ 011010 \\ 101001 \end{bmatrix}$$

Message vector (m)	Codeword (U)
000	000000
100	110100
010	011010
110	101110
001	101001
101	011101
011	110011
111	000111

Example: Block code (n,k)=(7,4)

$$G = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

m= [0 1 1 0]

↓

Linear Block Encoder (**U=m.G**)

↓

U= g_1+g_2

U= [1 0 0 0 1 1 0]

Message (m)	Codeword	
0000	0000000	
0001	**1010001**	g_3
0010	**1110010**	g_2
0011	0100011	
0100	**0110100**	g_1
0101	1100101	
0110	**1000110**	
0111	0010111	
1000	**1101000**	g_0
1001	0111001	
1010	0011010	
1011	1001011	
1100	1011100	
1101	0001101	
1110	0101110	
1111	1111111	

Example

$$\mathbf{G}=\begin{bmatrix}\mathbf{g}_0\\ \mathbf{g}_1\\ \mathbf{g}_2\\ \mathbf{g}_3\end{bmatrix}=\begin{bmatrix}1&1&0&1&0&0&0\\0&0&1&0&1&1&1\\1&1&1&1&1&1&1\\1&0&1&0&0&0&1\end{bmatrix}$$

(rows $\mathbf{g}_0$ to $\mathbf{g}_2$:) **Linearly Dependent**

m= [0 1 1 1]	**m**= [1 0 0 1]
Block Encoder (**U**=**m**.**G**)	Block Encoder (**U**=**m**.**G**)
U= $\mathbf{g}_1+\mathbf{g}_2+\mathbf{g}_3$	**U**= $\mathbf{g}_0+\mathbf{g}_3$
U= [0 1 1 1 0 0 1]	**U**= [0 1 1 1 0 0 1]

NOT DISTINCT

Tugas, Dikumpulkan !

Consider a (7,4) code whose generator matrix is

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

1. Find all the codewords of the code.
2. What is the error-correcting capability of the code?
3. What is the error-detecting capability of the code?

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